Rapport Seine-Aval 4

TIDEHYDREX

Forçages hydrologiques des aquifères du remplissage estuarien : un atelier expérimental pilote pour la prévision de l'influence des changements climatiques sur le fonctionnement hydrologique des zones humides.

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RAPPORT SCIENTIFIQUE

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I. Introduction :

Dans les zones humides du remplissage estuarien, la biodiversité et son évolution sont tributaires d'un fonctionnement hydrologique complexe qui est contrôlé par l'interaction hydrodynamique de nombreux compartiments : (1) les fluctuations tidales du niveau marin, (2) les fluctuations annuelles et pluriannuelles du débit du fleuve, (3) les fluctuations annuelles et pluriannuelles du gradient hydrologique de l'aquifère principal et de son bassin versant d'alimentation (localement, l'aquifère karstifié de la craie), (4) l'aquifère bi-couche du remplissage estuarien avec l'aquifère transmissif semi-captif des graves de fond surmonté par l'aquifère très peu transmissif du remplissage fin Holocène et, (5) les processus superficiels liés aux apports des pluies et de la gestion hydraulique du site qui n'étaient pas inclus dans cette proposition. Dans ce type de contexte, les modèles hydrologiques classiques basés sur les hypothèses de l'homogénéité des propriétés hydrauliques des aquifères sont loin d'êtres fiables pour modéliser les interactions entre les processus de recharge et de transfert de pression à l'interface entre domaines marin et continental.

Le site atelier proposé, dans les marais du Hode, bénéficie déjà d'un équipement, toujours fonctionnel, d'une série de piézomètres installés par le GPMH qui permet de disposer des fluctuations spatiales et temporelles des niveaux d'eau dans les aquifères superposés du remplissage estuarien. Dans le cadre d'un travail préliminaire réalisé, par notre équipe, pour le GPMH, le traitement statistique des chroniques piézométriques a révélé que les réponses hydrologiques de l'aquifère bi-couche du remplissage alluvionnaire enregistrent bien les forçages marins et d'écoulement de l'aquifère amont.

L'objectif principal de ce travail consistait à développer et tester de nouvelles approches pour caractériser les propriétés hydrauliques du sous sol pour établir un modèle numérique d'écoulements fiable en prenant en compte les échanges entre les différentes entités hydrogéologiques et les fluctuations des apports marins, localement caractérisées par les niveaux marégraphiques du chenal de Seine (données GPMR). La construction d'un tel modèle, adapté au contexte des marais du secteur d'étude (Figure 1), nécessite d'intégrer l'ensemble des données piézométriques disponibles et d'acquérir des données complémentaires géophysiques pour préciser les facteurs interactifs de l'état hydrique des sols. Ce type de modèle obtenu constituera alors un outil performant et évolutif pour répondre au questionnement de la gestion des contextes hydrologiques et environnementaux.

La méthodologie est basée sur une validation croisée entre les modèles hydrogéologiques et hydrogéophysiques voire hydrochimiques mais ce dernier volet n'a pas pu être testé dans le cadre de cette étude. Notre démarche scientifique a pour but d'apporter une compréhension quantitative et qualitative des différents processus hydrodynamiques du fonctionnement hydrologiques des aquifères côtiers.

Les simulations numériques d'écoulements souterrains sont principalement contrôlées par la distribution spatiale de la perméabilité hydraulique. La perméabilité peut varier sur plusieurs ordres de grandeurs comme elle peut être hétérogène à différentes échelles, ce qui génère des complexes modèles d'écoulements des eaux souterraines et du transport de contaminants. Pour cela, l'effort principal dans ce projet est porté sur la caractérisation spatiale des propriétés hydrauliques de l'aquifère semi-captif à partir de la réponse piézométrique au forçage tidale. Cette technique permet d'envisager la construction d'un modèle hydrologique multicouche qui pourra alors servir d'outil de gestion/protection/prédiction des ressources en eau côtières avec un bien plus grande fiabilité que les méthodes classiquement utilisées.
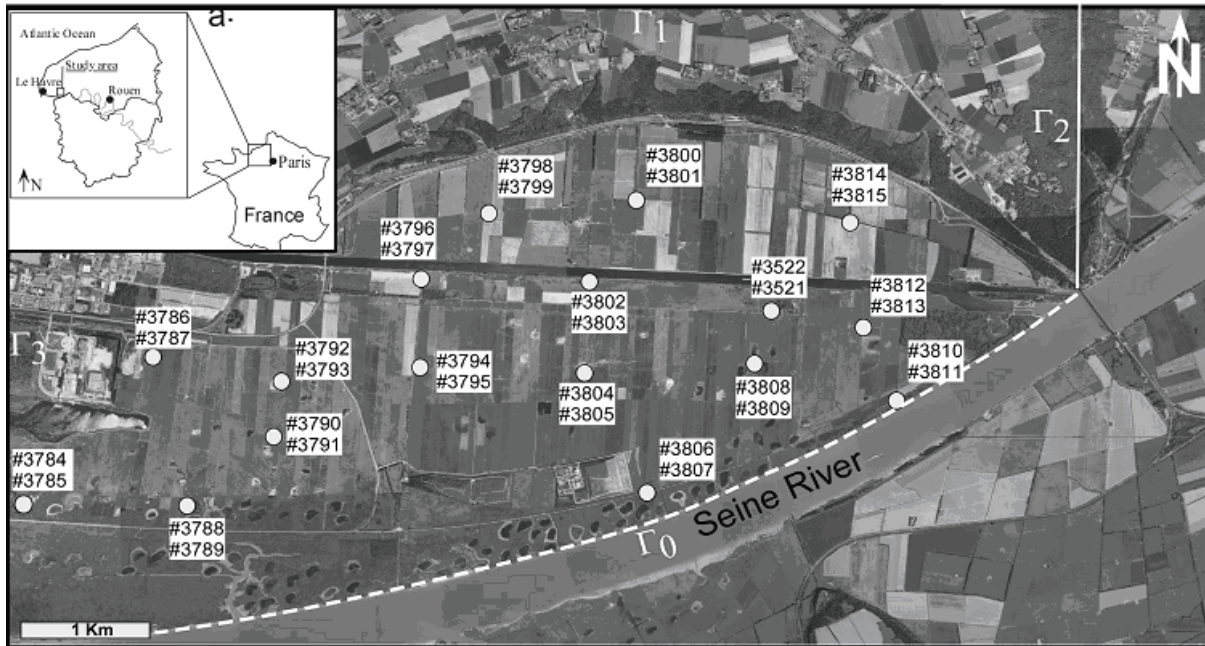
II. Contexte du site d'étude :

Alard et al (2002) ont décrit la diversité, l'intérêt écologique mais aussi la « fragilité » des zones humides de l'estuaire de Seine (Figure 1). Le Marais du Hode, localisé à proximité de l'embouchure correspond au site atelier choisi pour décrire le fonctionnement hydrologique de ces milieux d'interface.

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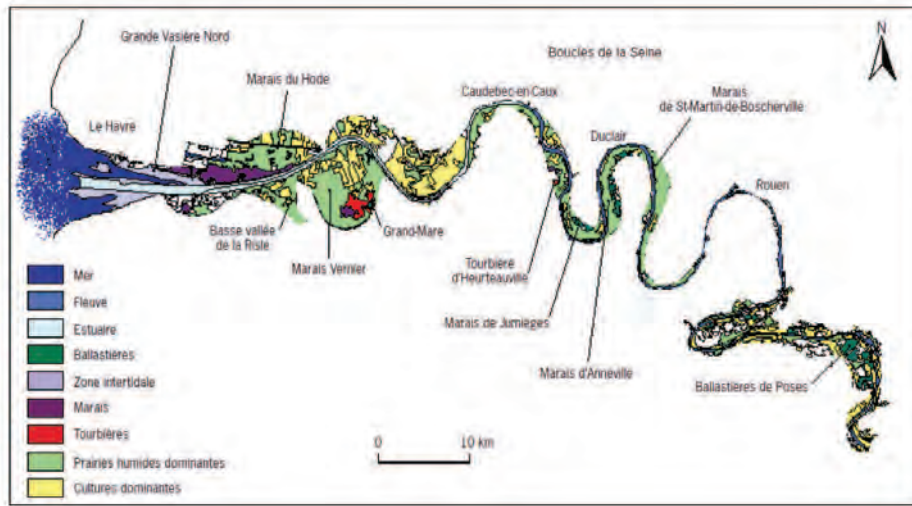
***Figure 1**.- Carte de localisation des principales zones humides de l'estuaire de Seine (d'après Alard et al, 2002)*

Le contexte géomorphologique et hydrogéologique du secteur d'étude est présenté dans la Figure 2. Il est limité, au Sud, par le chenal de navigation de la Seine dont les niveaux d'eau sont affectés par les fluctuations du débit de la Seine et un marnage très important (contexte mégatidal) pouvant atteindre localement 8,4 m. Ce chenal est limité par une digue submersible qui est symbolisée par un tiret blanc et à cette limite Γ_0 nous imposerons les fluctuations tidales comme contrainte hydrologique pour simuler numériquement les écoulements souterrains. Au Nord, la zone d'étude est bordée par une falaise de craie qui correspond à la limité Γ_1 qui représente l'apport de l'aquifère de la craie à la zone d'étude. A l'Est, Γ_2 les apports de l'aquifère karstique dans cette limité est négligeable. La limite Γ_3 représente l'interface d'échange hydrologique entre la partie Ouest des marais plus ou moins industrialisés et la zone d'étude.

Dans ce contexte, le GPMH a mis en place 17 ensembles de piézométrie (*cf* positionnement et références dans la Figure 2) comportant des piézomètres longs pour suivre la charge hydraulique de l'aquifère semi-confiné de la formation des graves de fond et des piézomètres plus courts pour étudier l'aquifère libre.

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Figure 2.- Carte de localisation du site d'étude avec la figuration des limites hydrologiques de la zone d'étude et le positionnement des 17 ensembles de piézomètres longs et courts dans la zone d'étude.

Les contextes géologiques et hydrogéologiques sont schématisés dans la Figure 3. A partir des données disponibles des carottages réalisés et profils géotechniques réalisés pour le GPMH, le remplissage estuarien est principalement constitué, à la base, de graves et sables graveleux perméables (bonne transmissivité hydraulique), surmontés par un niveau plus ou moins épais et, plus ou moins discontinu, de tourbes et matériaux fins très peu perméables (notés aquitard dans la Figure 3) puis, par une formation perméable de sables et silts qui est toutefois relativement moins transmissive que les formations graveleuses de la base de la série sédimentaire.

Les données recueillies dans les piézomètres longs, qui atteignent la formation graveleuse de base, révèlent que l'aquifère contenu dans cette formation est en charge sous l'aquitard figuré. Dans certains cas, les niveaux d'eau moyens de ces piézomètres sont mêmes supérieurs à ceux de l'aquifère libre de la formation supérieure des sables et silts (piézomètres courts). Localement, le niveau peu perméable, qui délimite les 2 entités hydrologiques, est quasi inexistant et permet des zones d'échange (essentiellement de l'aquifère semi-confiné vers l'aquifère non confiné, cf flèche 3 de la Figure 3). L'ensemble constitue donc un aquifère bi-couche qui est soumis, en amont, aux apports de l'aquifère karstique de la craie (limité à sa base par les argiles très imperméables du Gault, figurées dans la Figure 3) dont l'écoulement vers l'estuaire est contrôlé

par son gradient hydraulique amont/aval dont l'importance peut être mesurée au niveau des forages sous les plateaux du réseau patrimonial du suivi quantitatif des ressources en eau (ex. Piézomètre des Trois Pierres). Parallèlement, au Nord-Est de la zone d'étude, les fluctuations de débit de la rivière du Commerce, fournissent, pour la zone d'étude, une illustration partielle des fluctuations annuelles et saisonnières du forçage hydraulique de l'aquifère de la craie sur l'aquifère bi-couche étudié.

Dans ce contexte, les connaissances acquises par les auteurs permettent de rappeler que l'aquifère de la craie est un aquifère poreux fissuré drainé par les conduits karstiques bien développés en aval. Localement, ce contexte hydrologique est illustré par l'existence de l'ensemble des sources de Radicatel (forte contribution à l'alimentation en eau de l'agglomération du Havre), mais aussi de sources moins conséquentes et moins pérennes, en pied de falaise, dans la partie Nord du site d'étude. Ces exutoires karstiques correspondent à des sources karstiques dont le fonctionnement en surverse est lié au fait que l'aquifère non confiné des sables et silts présente une conductivité hydraulique plus faible que celle de l'aquifère de la craie. Les échanges entre ces 2 entités hydrologiques (flèche 5 de la Figure 3) sont donc limités.
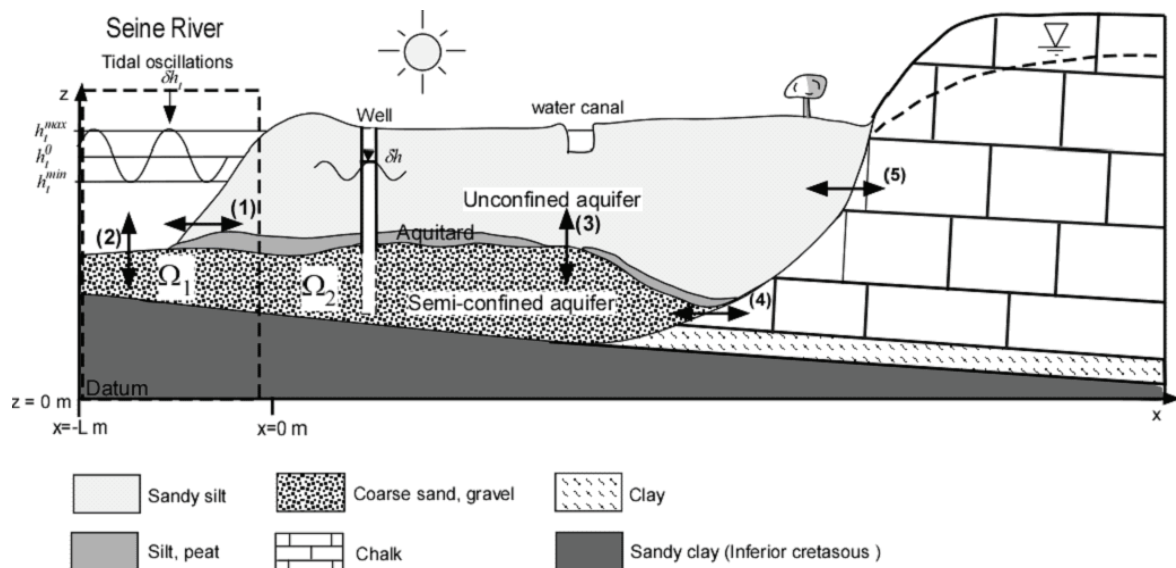
Figure 3.- Coupe schématique de l'organisation des entités géologiques et hydrogéologiques de la zone d'étude.

En revanche, la conductivité hydraulique des graves de fond (estimations quantitatives incluses dans la suite de ce rapport) est globalement proche de celle mesurée en moyenne dans l'aquifère de la craie (regroupement des écoulements poreux fissurés et des écoulements karstiques), à

l'approche des vallées : de l'ordre de 10^{-3} à 10^{-2} m.s^{-1}). Cette convergence des propriétés hydrauliques de ces 2 entités permet d'assurer le principal continuum hydrologique entre l'aquifère de la craie, en amont et, la nappe semi confinée des formations graveleuses de la base du remplissage estuarien, en aval (flèche 4 de la Figure 3). Hydrologiquement, ce point fondamental du système hydrologique complexe étudié est confirmé dans les parties suivantes de ce rapport.

Latéralement, en progressant du Nord au Sud, la formation graveleuse de fond repose sur des formations géologiques plus anciennes du Crétacé inférieur et du Jurassique (cœur de l'anticlinal de l'estuaire de Seine) qui correspondent à des alternances entre des formations imperméables et perméables et constituent donc des aquifères captifs en charge dont la puissance, la capacité d'emmagasinement et le potentiel de réalimentation sont beaucoup plus limités par rapport aux apports de l'aquifère de la craie. Dans cette étude, nous considérons donc comme négligeables le flux de ces remontées d'eau à partir des aquifères plus profonds (absence de flèche d'échange hydrologique dans la Figure 3).

Vers le Sud de la zone d'étude, les écoulements de l'aquifère bi-couche du remplissage estuarien sont contraints par les fluctuations des niveaux d'eau en Seine qui sont inféodées à l'influence des marées de régime macrotidal ainsi qu'aux variations de débit de la Seine. Pour l'aquifère non confiné, les échanges hydrologiques avec la Seine sont figurés par la flèche 1 de la Figure 3.

En dépit des dépôts récents au fond du chenal endigué de la Seine, le creusement et les dragages d'entretien de ce chenal de navigation favorisent les échanges hydrologiques avec l'aquifère semi confiné (flèche 2 de la Figure 3).

III. Détermination des contributions respectives de l'aquifère de la Craie et du niveau de la Seine dans les variations piézométriques du remplissage alluvionnaire.

Une analyse par ondelettes continue a été appliquée sur des séries temporelles de données piézométriques des deux aquifères pour identifier les facteurs de contrôle de la variabilité piézométrique. Notre recherche ne se limite pas à l'identification des périodicités liées au régime tidal, mais également d'une variabilité liée à la contribution de processus très certainement non-stationnaires (variations piézométriques de la nappe de la Craie), a justifié l'emploi de cette approche méthodologique.

Les diagrammes temps-fréquence des piézomètres tant courts (alluvions fines) que longs (graves de fond) montrent dans tous les cas la présence caractéristique d'une bande d'énergie typique du signal tidal de période d'environ 12h, en outre clairement modulée par une périodicité

à 14j, comme cela pouvait être observé à la source karstique du Hannetot. Dans le cas des graves de fond, le diagramme temps-fréquence montre que la variation tidale est la composante dominante de la variabilité piézométrique (Fig.4). Cet impact tidal perdure malgré un effet notable de la distance à la Seine : la part de variance calculée pour les variations liées au marnage explique pour les piézomètres implantés dans les graves de fond de forte transmissivité de 39% à plus de 97% de la variabilité piézométrique totale selon l'éloignement du piézomètre par rapport à la Seine.
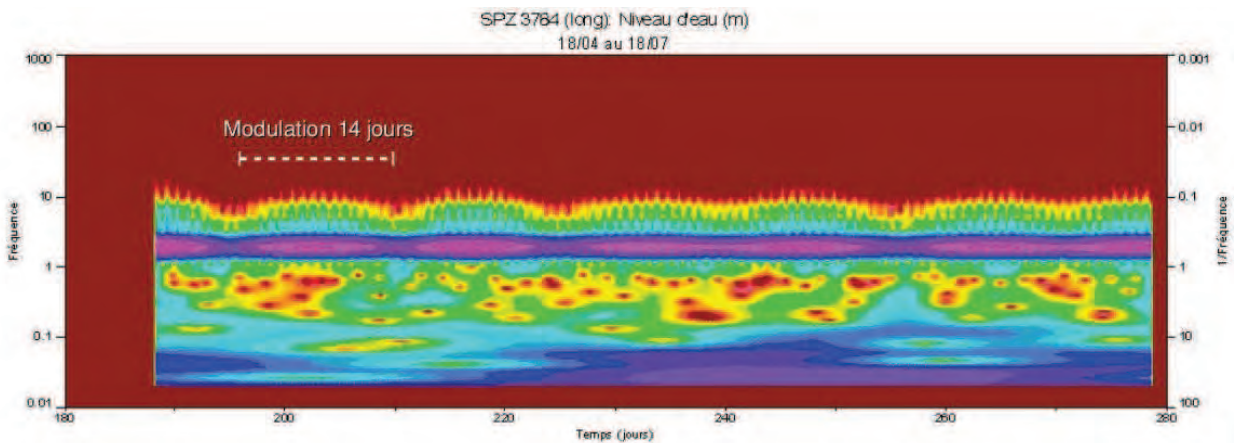
Figure 4. *Spectre en ondelettes d'un signal piézométrique pour un piézomètre long #3784 (graves de fond,).*

Dans les alluvions fines (piézomètres courts), l'effet du marnage tidal, bien que présent, est fortement atténué, sauf pour les piézomètres localisés tout à fait au bord de la Seine où il reste conséquent. L'essentiel des variations piézométriques dans les alluvions fines s'apparentent aux variations, connues par ailleurs, de la charge dans l'aquifère karstifié de la Craie (Fig.5).
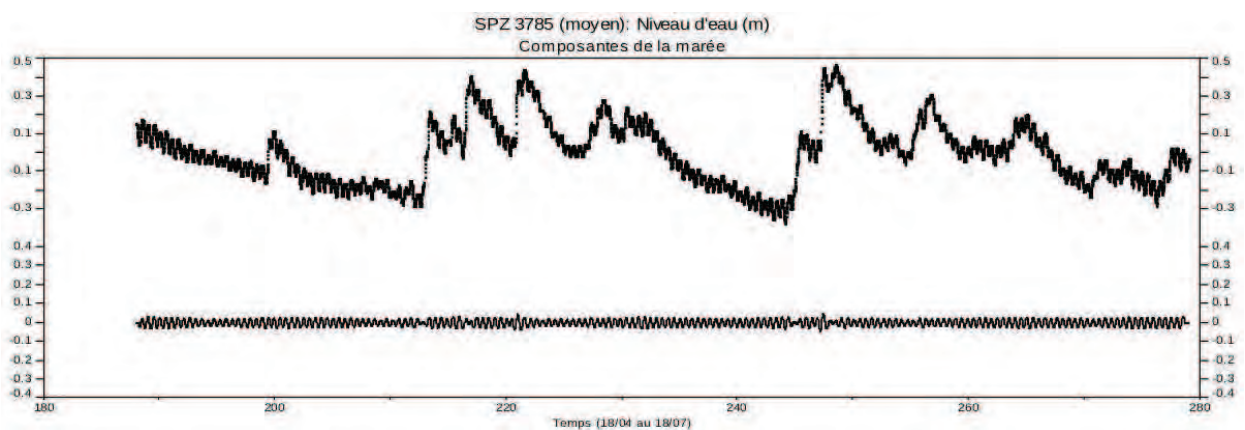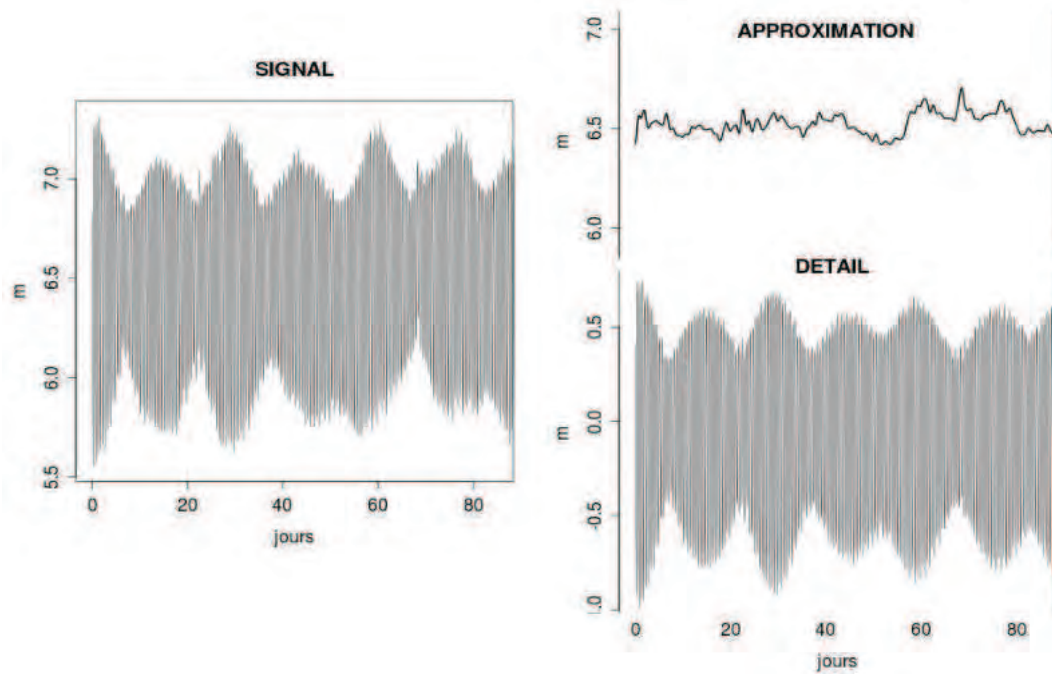

Figure 5. *Charge hydraulique pour un piézomètre court #3785(alluvions fines du supérieures, en haut) et composante tidale extraite par transformée en ondelettes (en bas).*

Une analyse multirésolution sur les piézomètres des graves de fond permet l'extraction des différentes composantes du signal piézométrique : il est ainsi possible d'isoler la part de variabilité liée au marnage en Seine (détail de la décomposition multirésolution, Fig.6), ainsi que la présence de variations de plus grande échelle temporelle (approximation, Fig.6). L'analyse montre que même si la piézométrie dans les graves de fond est largement dominée par les cycles tidaux, les fluctuations basse-fréquence (approximation, Fig.6) sont manifestement structurées de manière analogue aux variations piézométriques des alluvions fines du dessus, c'est à dire à la manière des réponses karstiques de l'aquifère de la Craie.

Dans tous les cas, quelque soit le piézomètre considéré, la composante basse-fréquence prend l'allure d'hydrogrammes de crue, manifestement en lien avec la nature localement karstique de la Craie sous-jacente. Une comparaison avec les hydrogrammes de deux bassins versants superficiels majeurs localisés en bordure de plateau crayeux de part et d'autre de la zone étudiée montre une bonne adéquation des différentes réponses hydrologiques à la succession des événements pluvieux (Fig.7). Une telle réactivité dans l'ensemble alluvionnaire correspondrait donc à l'expression de l'aquifère de la Craie karstifiée sous les alluvions. Ces nouvelles analyses ont mis en évidence l'importance de l'influence tidale dans les piézomètres longs mais surtout l'importance d'une influence « continentale » dans les piézomètres courts. Cette influence continentale pourra être déterminée de manière plus précise lorsque nous disposerons des données de niveau d'eau et de conductivité électrique dans les piézomètres courts. En effet, le fonctionnement hydraulique de surface est différent du fonctionnement hydraulique souterrain, ces piézomètres courts nous permettrons donc de distinguer les forçages de types continentaux des forçages de types estuariens pour l'ensemble de la zone.

Figure 6. *Analyse multirésolution de la charge hydraulique dans un piézomètre long (graves de fond), faisant apparaître pour les grandes échelles de la décomposition (approximation) des variations liées à la réponse hydrologique de l'aquifère karstifié de la Craie.*
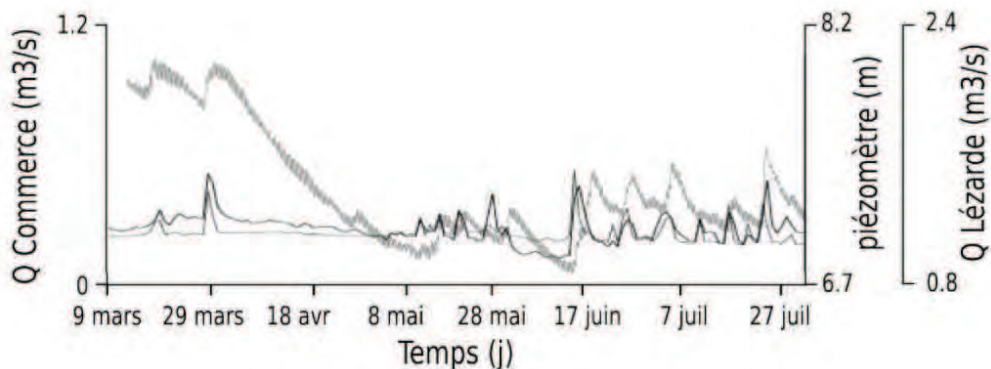
Figure 7. *Comparaison entre fluctuations piézométriques basse-fréquence du remplissage alluvionnaire et réponses hydrologiques de 2 rivières encadrant la zone étudiées.*

Les résultats présentés dans ce rapport confirment l'importante hétérogénéité des marais du Hode dans son fonctionnement hydraulique. Cette hétérogénéité posera problème sans doute lors de la phase de modélisation, notamment à cause des conduits karstiques à proximité des piézomètres #3804 et #3805 (figure 2). Ces informations doivent être prises en compte pour établir un modèle numérique numérique des écoulements souterrains.

VI.- La Tomographie hydraulique de l'aquifère semi-confiné

1.- Démarche analytique du problème (*voir l'annexe A pour plus de détails*)

La gestion des ressources en eau nécessite une connaissance précise de la géométrie des aquifères et de la distribution de leurs propriétés physiques, en particulier la perméabilité hydraulique. La méthode classique de caractérisation de la transmissivité hydraulique (perméabilité hydraulique multipliée par l'épaisseur de l'aquifère) et le coefficient d'emmagasinement est basée sur les essais de pompage (Theis, 1935). Cette dernière n'offre qu'une estimation locale de ces paramètres en question. Récemment, la tomographie hydraulique a été développée afin de caractériser l'hétérogénéité des propriétés hydrauliques des aquifères à partir de l'inversion des variations piézométriques collectées lors des tests de pompage. Cette technique a été validée à la fois dans une échelle réduite avec des expériences en laboratoire (Liu et al, 2002, 2007.) et, dans des conditions de terrain, Straface et al (2007). Cardiff et al. [2009] ont utilisé l'approche quasi-linéaire de la géostatistique développée par Kitanidis (1995) pour cartographier la distribution de la transmissivité hydraulique du Site de Recherche Hydrogéophysique de Boise (BHRS). D'autres hydrogéologues ont proposé une autre alternative appelée la méthode des points pilotes pour reconstruire le champ de la transmissivité hydraulique. Cette méthode réduit le nombre des paramètres à estimer, en identifiant en quelques-endroits de l'aquifère les valeurs de la transmissivité hydraulique avec le processus de l'inversion. Le champ de la transmissivité du domaine étudié est obtenu par le krigeage de la transmissivité hydraulique des points pilotes. Dans cette approche, les valeurs et les positions des points pilotes sont utilisés pour perturber le champ de la transmissivité d'un modèle a priori qui est le résultat du krigeage des données observées de la transmissivité. Les valeurs et les positions optimales des points pilotes sont traditionnellement déterminées par un algorithme d'optimisation déterministe pour reconstruire les variations piézométriques observées. Par conséquent, les valeurs et les emplacements des points pilotes sont considérés comme paramètres de calibration. Ce processus, associant les caractéristiques géostatistiques des aquifères (comme les longueurs de corrélation) et les données piézométriques, a été appliqué avec succès à divers problèmes hydrogéologiques (Lavenue et Pickens, 1992; Lavenue et al, 1995; RamaRao et al, 1995).

Afin de réduire l'instabilité liée au choix de l'emplacement des points pilotes et de l'hyper-paramétrisation du problème, un certain nombre d'approches ont été proposées au cours des deux dernières décennies sur la façon de choisir les positions des points pilotes (Certes et de Marsily , 1991; Lavenue et al, 1995;. RamaRao et al, 1995; Cooley, 2000; Medina et Carrera, 2003; Doherty, 2003; et al, 2005 Carrera).

Dans ce projet, nous sommes intéressés par l'imagerie du champ de transmissivité hydraulique d'un aquifère alluvionnaire semi-confiné connecté avec l'estuaire d'une rivière sous l'influence tidale. Généralement, dans le cas des aquifères côtiers, les essais de pompage ne sont pas toujours souhaitables, car ils peuvent accélérer l'intrusion marine (Chapuis et al., 2006). Comme le niveau hydraulique des aquifères côtiers fluctue naturellement avec les oscillations tidales, il est opportun d'utiliser ces fluctuations naturelles de niveau piézométrique comme une source de perturbation à la place de pompage pour estimer les paramètres hydrauliques (Jacob, 1950; Ferris, 1951 et Jiao et Tang, 1999). Bataille-Aguiar, (2008) a combiné des tests de pompage / injection, des expériences de traçage et les enregistrements de l'interaction des eaux souterraines / rivière pour caractériser les propriétés du transport d'une nappe alluviale. Dans ce que l'on appelle «la réponse tidale», les hydrogéologues analysent les fluctuations de charge hydraulique d'un ensemble de piézomètre en réponse aux changements de niveau marin causé par les marées (Jacob, 1950; Ferris, 1951; Gregg, 1966; Carr et van der Kamp 1969 et Nielsen, 1990). Cette approche nécessite donc de disposer des enregistrements marégraphiques et piézométriques enregistrés dans un ou plusieurs piézomètres affectés par les variations de la marée. En utilisant ces informations, la modélisation inverse est utilisée pour estimer les propriétés hydrauliques (càd la conductivité hydraulique où le coefficient de diffusion) des aquifères côtiers (Jiao et Tang, 1999 et Jeng et al., 2002). Ce processus d'interprétation est basé sur une équation analytique simple 1D qui décrit la réponse des eaux souterraines d'un aquifère homogène et isotrope à des oscillations tidales imposées comme conditions aux limites (Todd, 1959). Cette solution montre que la signature de niveau hydraulique décroît d'une façon exponentielle avec la distance (piézomètre-cote). La décroissance exponentielle est appelée le facteur d'efficacité tidale. Les variations piézométriques restent cycliques mais avec un déphasage par rapport aux oscillations. Le facteur d'efficacité tidale et le déphasage dépendent du rapport de la transmissivité et du coefficient d'emmagasinement de l'aquifère étudié (Serfes, 1991; Soleil, 1997). Par conséquent, ce protocole ne fournit qu'une diffusivité hydraulique moyenne des aquifères côtiers en raison de l'utilisation de l'hypothèse d'homogénéité. Récemment, Alcolea et al. (2007, 2009) ont présenté une tentative rigoureuse basée sur la méthode des points pilotes pour cartographier le champ de la transmissivité hydraulique d'un aquifère côtier non confiné avec la combinaison des variations piézométriques liées aux oscillations tidales et aux tests d'injection.

Notre démarche vise à étendre cette approche au cas de deux aquifères connectés à l'estuaire de la Seine soumis à l'influence tidale. Un de ces aquifères est un aquifère semi-confiné, qui est connecté localement avec un aquifère libre de surface. En plus de la

reconstruction de la transmissivité hydraulique; nous cherchons à quantifier les échanges et flux hydrauliques entre les deux aquifères à partir des réponses piézométriques aux variations tidales. L'approche proposée prend également en compte l'impact des écoulements souterrains de l'aquifère karstique de la craie sur les fluctuations des données piézométriques du remplissage alluvionnaire. Tandis que les précédentes études utilisaient des méthodes déterministes pour estimer les valeurs des points pilotes, nous avons choisi de distribuer les points pilotes sur un maillage régulier et nous avons utilisé une méthode stochastique (basée sur un algorithme de chaîne de Markov Monte Carlo, MCMC) pour prendre en compte de la non unicité de la solution du problème inverse.

Notre travail a aussi pour ambition de traduire la complexité du fonctionnement hydrologique du site d'étude en intégrant trois nouveaux aspects : (1) nous avons affaire à deux aquifères avec le facteur de drainage entre l'aquifère semi-confiné et l'aquifère libre, (2) nous avons pris en considération les apports des écoulements de l'aquifère karstique qui est la principale source d'alimentation des deux aquifères (3), nous avons développé un processus stochastique plutôt qu'un déterministe pour reconstruire le champ de la transmissivité hydraulique.

2.- La tomographie de la transmissivité hydraulique :

Les variations piézométriques de l'aquifère semi-confiné enregistrées sur les 17 piézomètres comprennent à la fois l'effet de l'écoulement des eaux souterraines et les fluctuations harmoniques dues aux oscillations de marée. Les données piézométriques sont enregistrées au cours d'une période de 100 heures pendant laquelle nous pouvons considérer que les apports de l'aquifère karstique sont demeurés stables ainsi que l'infiltration verticale. L'effet de la circulation régionale sur les niveaux piézométriques dans la zone alluviale Ω_a peut être considéré comme la valeur moyenne des données piézométriques dans la fenêtre de temps considérée (Figure 8). Les données disponibles dans le domaine karstique au Nord de la zone d'étude ont permis d'imposer une charge hydraulique de 8 m à la limite Nord du domaine karstique, avec une transmissivité de 10^{-3} m.s^{-2} (Hanin [2010]). Afin de déterminer la réponse hydraulique associée aux oscillations de marée et d'éviter d'intégrer les effets de la circulation hydrique régionale horizontale, nous avons corrigé les données de charge hydraulique de la circulation hydrique régionale. Cela a été fait pour chaque forage en soustrayant la valeur moyenne au cours du temps afin de travailler plutôt sur les fluctuations que sur les valeurs elles mêmes.
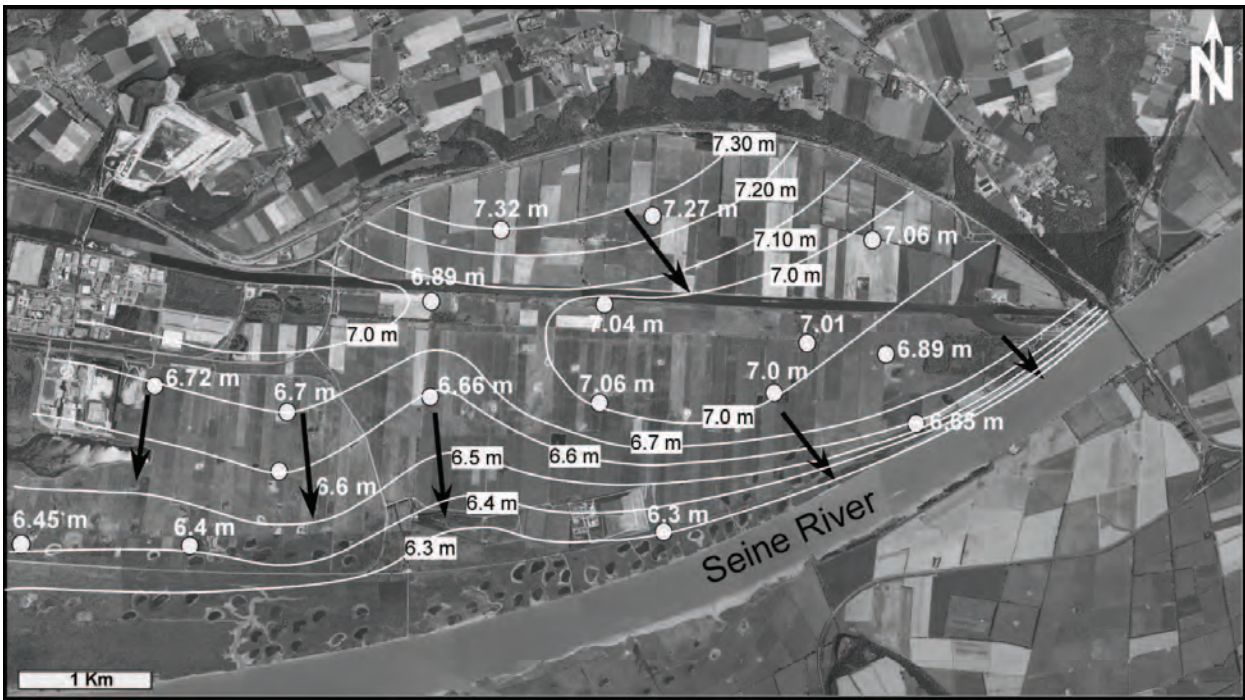
Figure 8. *La distribution spatiale du niveau hydraulique régional due aux apports de l'aquifère karstique*

L'effet de la réponse résiduelle liée à l'effet de marée est illustré dans la figure 9 dépend principalement : (1) de la distance à la rivière, (2) des valeurs de diffusivité hydraulique de l'aquifère locales et (3) des échanges hydrauliques entre les aquifères semi-confiné et non confiné en fonction de la puissance de l'aquitard intermédiaire. Le gradient Nord-Sud illustre l'aspect distance à la rivière mais, dans ce contexte, la différence de l'amplitude des fluctuations cycliques de charge hydraulique au niveau de forages situés à la même distance du fleuve illustre l'influence de l'hétérogénéité de la transmissivité de l'aquifère semi-confiné. Dans les chroniques piézométriques, nous constatons que l'amplitude des variations diminue rapidement avec la distance, dans la partie centrale de la zone étudiée (piézomètres # 3802, # 3804, et #3806). Inversement, dans la partie ouest du site (piézomètres # 3784, # 3786, # 3788, # 3790, # 3792 et), le signal de marée est relativement moins atténué (Figure 9).
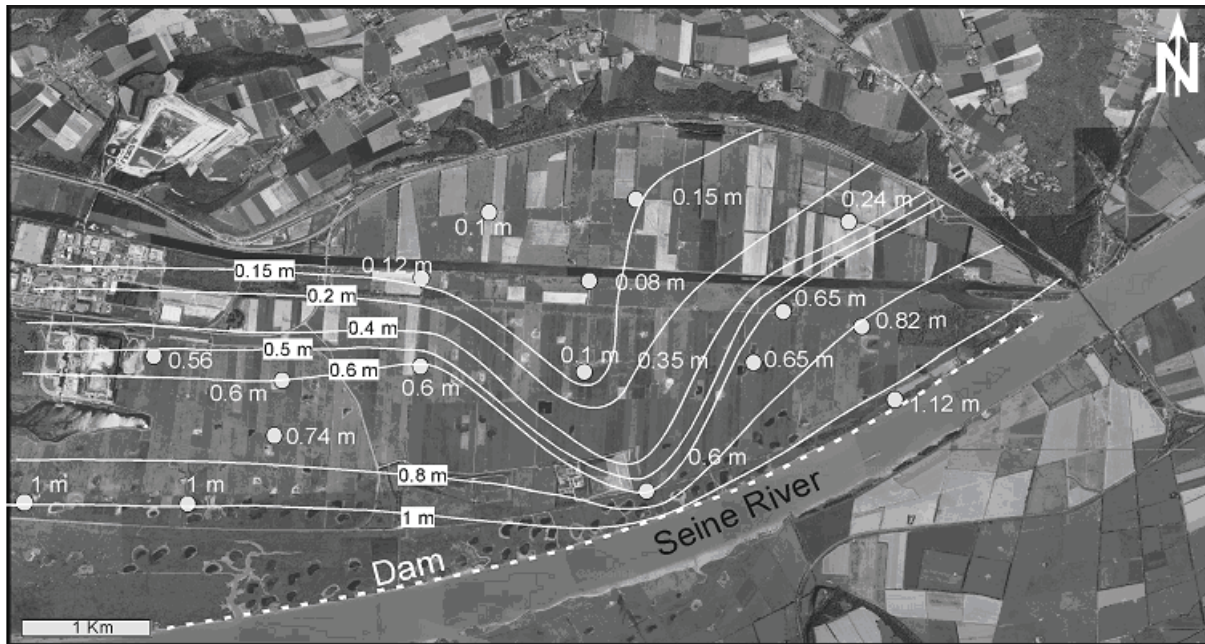
Figure 9. *Carte de l'amplitude des fluctuations de marée dans le site d'étude. La distribution spatiale obtenue montre que l'amplitude varie en fonction* (1) *de la distance à la rivière,* (2) *des valeurs de diffusivité hydraulique de l'aquifère et* (3) *des échanges hydrauliques entre les aquifères semi-confiné et non confiné. La ligne pointillée, notée Dam, correspond à une digue quasi non submersible dans la partie Est de la zone et à une digue submersible dans la moitié Sud-Ouest de la zone d'étude qui est dédoublée (quelques centaines de mètres plus au Nord) par le prolongement de la digue non submersible Est. Cette dernière délimite les zones de vasières intertidales des marais côtiers de la zone d'étude. Ces digues sont construites dans les dépôts de surface sur la rive du fleuve et, constituent une frontière hydraulique avec des charges hydrauliques différentes de part et d'autre.*

Dans le contexte de la frange Sud du site d'étude, les digues constituent une frontière hydraulique entre le chenal de Seine soumis directement à l'influence des fluctuations tidales et les marais. Cette limite est simple dans la partie Sud-Est avec la digue qui constitue la barrière entre chenal et les marais étudiés. Dans la partie Ouest du secteur d'étude, la digue du chenal est submersible avec une zone de vasières intertidales qui est limitée par la digue qui protège les marais du Hode (zone d'étude) de l'inondation tidale. Quoi qu'il en soit, le système des digues délimite au Sud le secteur du chenal qui est le lieu de l'application verticale du forçage tidal maximum qui est transmise à l'aquifère semi confiné sous-jacent. Compte tenu de la perméabilité de cette formation, le transfert de pression se transmet ensuite horizontalement sous le compartiment marais et crée localement un contraste fort entre les aquifères semi confiné et non confiné en matière de fluctuation des charges hydrauliques liées à la marée. L'influence hydraulique de la digue est figurée par une zone dans laquelle la charge hydraulique de l'aquifère libre devient inférieure à celui de l'aquifère semi-confiné. Par conséquent, le forçage vertical devient un facteur important à quantifier dans l'analyse hydrogéologique.

Dans ce domaine, les chroniques piézométriques enregistrées à la fois dans l'aquifère non confiné et dans l'aquifère semi-confiné indique une présence significative d'un drainage vertical vers le haut. Par exemple, la figure 10a montre que la charge hydraulique moyenne de l'aquifère libre (# 3811) est nettement plus faible que celle de l'aquifère semi confiné (# 3810). Par conséquent, le forçage vertical entre les deux aquifères est fort et est responsable des fluctuations de marée, relativement plus fortes, observées ici tant dans l'aquifère non confiné que dans l'aquifère libre (figure 10a). En outre, cette notion de forçage vertical se retrouve dans une zone où la présence de l'impact du barrage n'a pas d'influence, comme indiqué par les variations piézométriques aux forages # 3802 et # 3803 (voir Figure10b). Dans ce cas, le forçage vertical ou fuite verticale ne crée pas une augmentation de la charge hydraulique de l'aquifère libre (voir la figure 10b). Par conséquent, le forçage entre les deux aquifères est lié aux apports de l'aquifère de la craie (considérés localement comme très peu soumis à l'influence des marées), ce qui n'est pas éliminé par le filtrage des données piézométriques. Cela démontre que l'effet d'un écoulement vertical doit être pris en compte dans notre modélisation. Li et al. (2002) ont montré que la fuite d'un aquifère semi-confiné dans un aquifère sus-jacent en milieu ouvert diminue de manière significative la propagation de la marée. Plusieurs auteurs ont utilisé des modèles analytiques pour examiner un tel effet sur l'estimation de la diffusivité hydraulique $D = T / S$ (Li et al, 2001; Jiao et Tang, 1999).
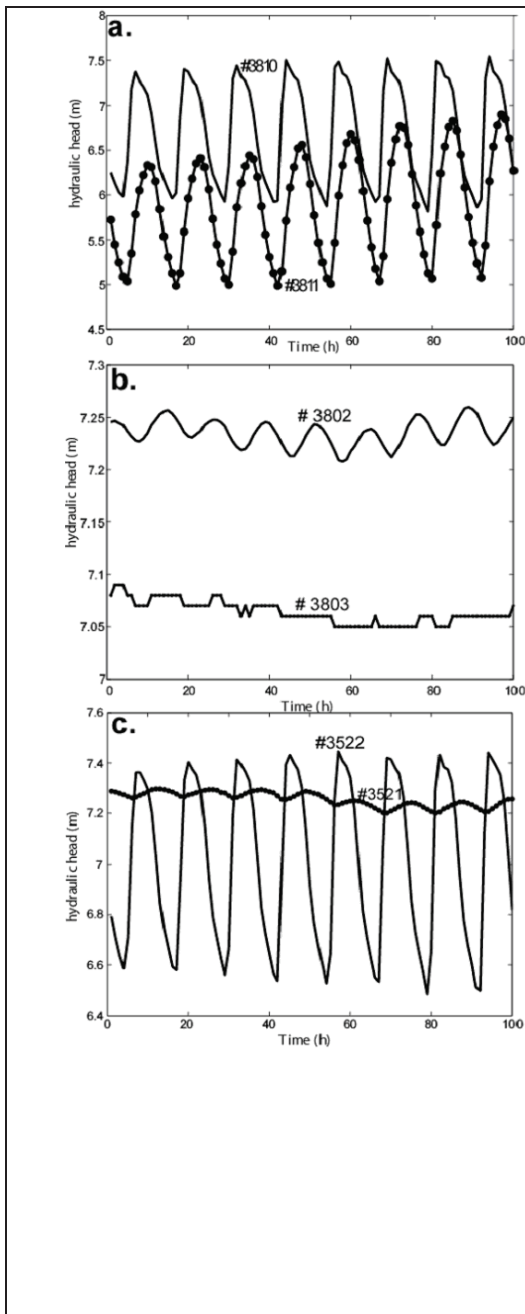
Figure 10. Quelques hydrogrammes enregistrés dans les aquifères semi-confinés et non confinés dans trois zones distinctes soumises à l'influence du forçage vertical de l'aquifère semi-confiné vers l'aquifère non confiné. Dans ces trois domaines, la charge hydraulique de l'aquifère libre devient inférieure à celle de l'aquifère semi-confiné. Les piézomètres # 3810 et # 3810 montrent l'importance du flux vertical associé à la présence des digues. Les piézomètres # 3802 et # 3803 montrent le comportement de la charge hydraulique de la zone où le flux vertical est associé au forçage latéral de l'écoulement de l'aquifère de la craie. Il n'y a pas de variations significatives de la charge hydraulique dans le piézomètre # 3803 ce qui y traduit une forte atténuation du signal de marée. Les piézomètres # 3522 et # 3522 correspondent à une zone où l'infiltration verticale est présente de façon transitoire seulement lors des grandes marées.

Nous cherchons donc à introduire, localement, un coefficient de distribution pour traduire le processus des fuites ou écoulements verticaux (forçage accru et flux de l'aquifère semi confiné vers l'aquifère non confiné, notamment lorsque l'aquitard est moins efficient) dans le processus de modélisation inverse. Nous quantifions ces flux hydriques seulement dans les zones où la fuite a été enregistrée dans les piézomètres. Dans ce cadre, nous utilisons une démarche de zonage comme méthode de paramétrisation au lieu de l'approche des points pilotes en raison de la difficulté de l'estimation d'un variogramme expérimental pour le coefficient d'infiltration verticale. Dans notre algorithme, ces estimations sont déterminées en même temps que l'estimation de la transmissivité dans les points pilotes. Enfin, l'approche McMC est utilisée pour déterminer les propriétés hydrauliques en générant, à chaque itération, deux modèles. L'une de

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ces deux modèles correspond à la perturbation du champ de transmissivité $f_P(\mathbf{x})$ qui est ajoutée au champ de transmissivité initial afin de déterminer un champ de transmissivité optimal qui reproduit les observations piézométriques présentées en cercles pleins dans la figure 11.
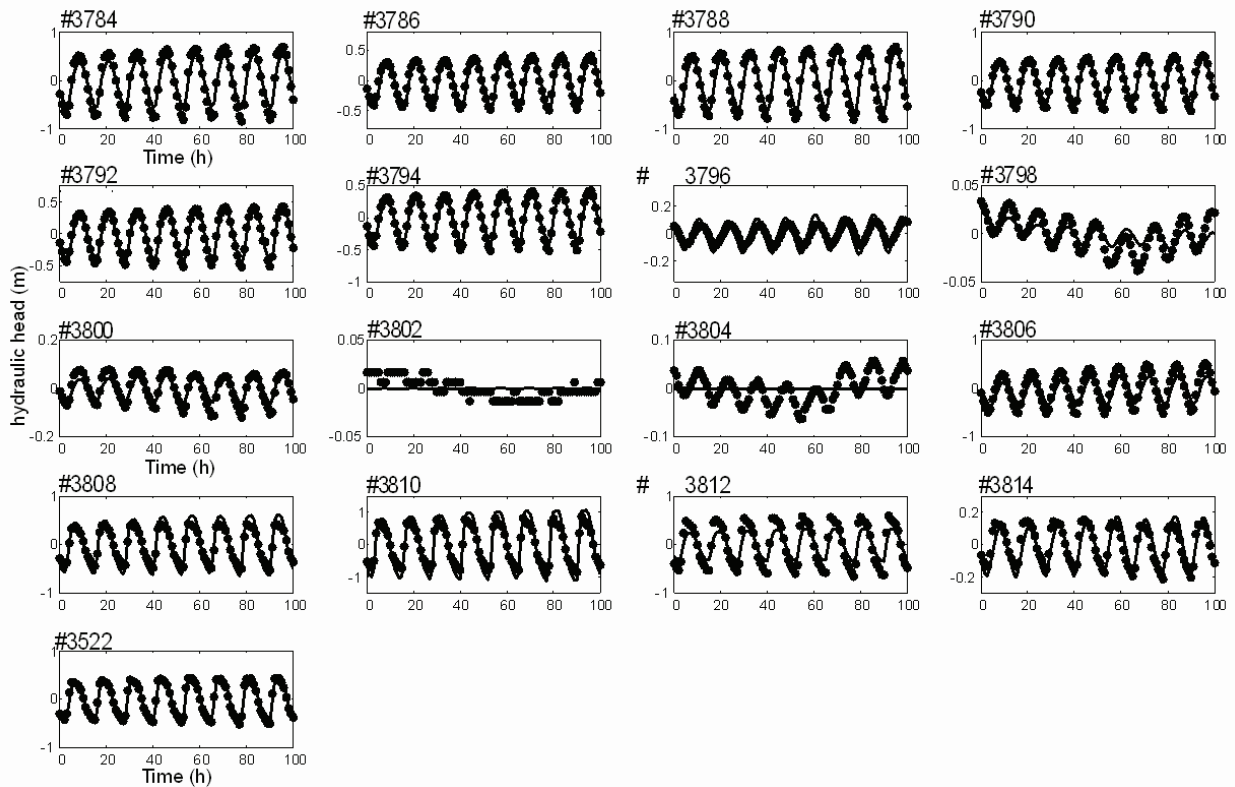
Figure 11. *Comparaison entre les hydrogrammes observés (cercles pleins) et inversés (lignes simples) dans l'aquifère semi-confiné.*

Les informations géologique, hydrogéologique et géotechnique existent dans un ensemble de forages et comprennent des données de tests géotechniques et un test de pompage hydraulique mené sur le forage # 3794. Ces informations combinées sont utilisées pour construire un champ initial de la transmissivité observée (Figure 12a). Le champ de transmissivité initial est obtenu par krigeage des valeurs observées de $\text{Log}_{10}T$ en utilisant l'approche ordinaire du krigeage avec un variogramme expérimental sphérique et isotrope avec une longueur de 3000 m et une variance de 1 log10[m2 s^{-1}] ^{2}. Ce champ initial sera perturbé par les points pilotes dans chaque itération durant le processus de l'inversion afin de reconstruire les observations piézométriques. Par conséquent, la performance de la modélisation d'inversion est fortement attribuée à la perturbation du champ généré par les points pilotes. Nous avons choisi dans cette étape une distribution dense et régulière de 72 points pilotes (Figure 12b). Les valeurs

des points pilotes sont estimées grâce à l'optimisation stochastique de la fonction objective. Dans ce projet le coefficient d'emmagasinement est supposé constant sur tout le domaine (S = 10-5 selon un essai de pompage effectué sur le forage # 3794). L'optimisation est réalisée avec l'algorithme *Adaptive Metropolis Algorithm*. Un total de 30.000 itérations sont utilisées pour reconstruire le champ de perturbation et les débits de drainage entre les deux aquifères (le champ de la transmissivité obtenu est reporté sur la figure 12b et pour l'évaluation de la composante de drainage vertical voir la figure 13b). Le champ de perturbation a été fourni à partir du krigeage de la moyenne des valeurs de points pilotes échantillonnées durant le processus itératif d'inversion.
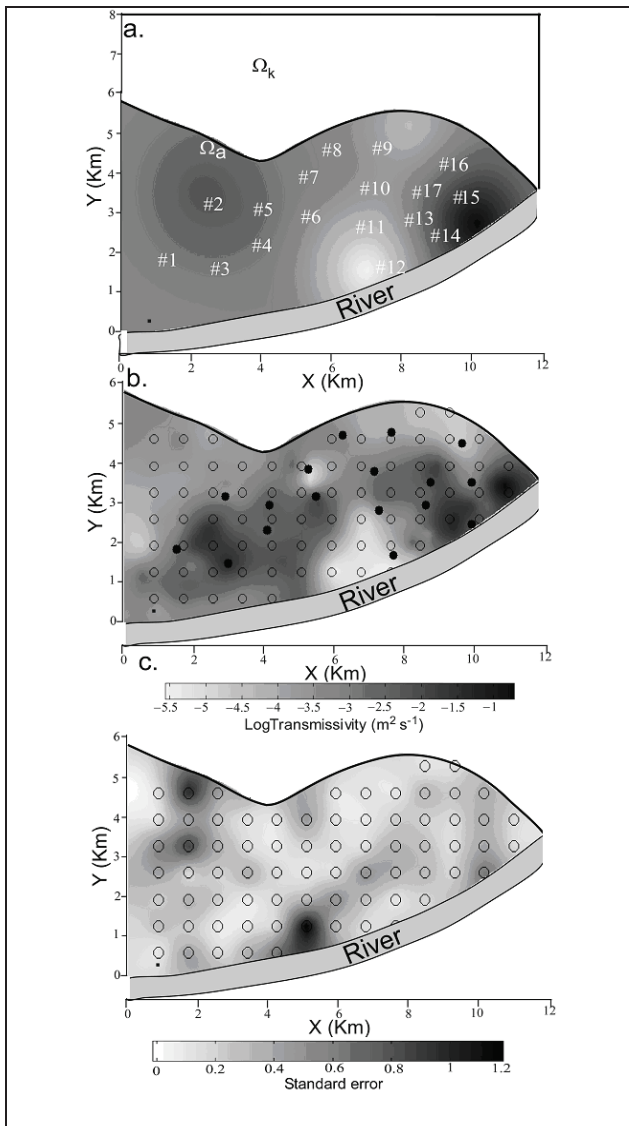
Figure 12.- *Champ de la transmissivité pour le cas d'étude.* **a**. *le champ initial de la transmissivité est obtenu en utilisant le krigeage ordinaire des mesures de $\log_{10}T$.*

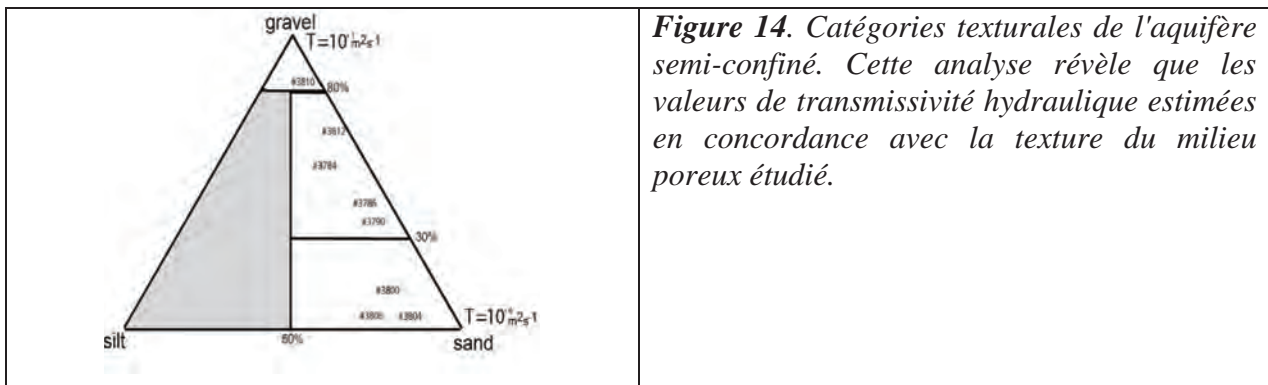**b**. *Le champ de la transmissivité hydraulique estimé par l'inversion des fluctuations*
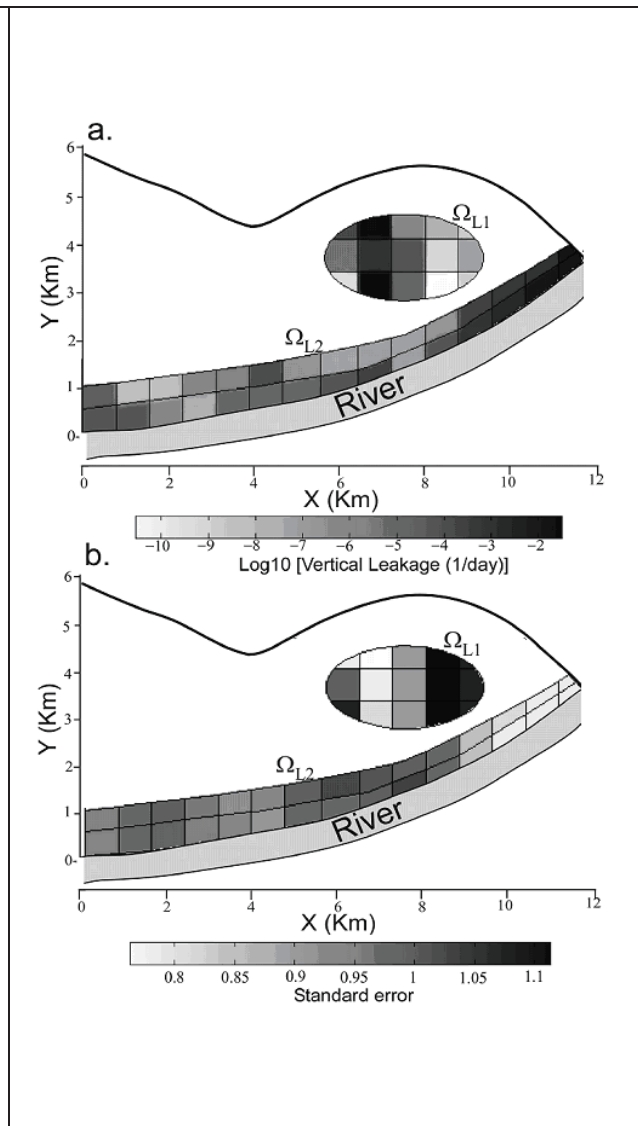
Figure 13.- *Estimation de la distribution du coefficient L de forçage vertical de l'aquifère semi confiné vers la nappe libre. Les zones Ω_{L2} et Ω_{L1} correspondent à des zones où le forçage vertical est considéré comme significatif. La zone Ω_{L1} correspond à un*

piézométriques. Les cercles ouverts présentent les positions des points pilotes. Les cercles noirs correspondent à l'emplacement des piézomètres. c. Distribution de l'erreur standard du champ de transmissivité estimé.	*domaine dans lequel le forçage vertical est probablement lié aux apports latéraux de l'aquifère de la craie. La zone Ω_{L2} correspond à une zone où le forçage vertical est généré par l'influence hydrologique du système de digue séparant les secteurs des marais et du chenal de la Seine.*

La variabilité spatiale du champ de transmissivité (Figure12b) révèle que l'aquifère semi-confiné est hétérogène et très perméable avec des valeurs de transmissivité variant de 10^{-1} à 10^{-5} m^2 s^{-1}. Nous vérifions que le champ inversé est compatible avec les observations indépendantes.

(1) Une zone de faible transmissivité ($<10^{-4}$ m^2 s^{-1}) est située au pied de falaise, au Nord de la zone d'étude (Figure 12b). Cette faible transmissivité fournie par inversion est compatible avec la présence de sédiments mal triés avec des sables fins et argiles limoneuses conforme aux analyses sédimentologiques réalisées dans les forages (Figure 14). En outre, une contribution de ces sédiments limoneux et argileux est également été mentionnée dans les forages # 3804 et # 3806.

(2) Les valeurs de transmissivité élevées déduites de l'inversion pour l'aquifère semi-confiné sont également compatibles avec la présence de dépôts alluviaux dominés par des graviers et de sables (Figure 12b et Figure 14).

	Figure 14. *Catégories texturales de l'aquifère semi-confiné. Cette analyse révèle que les valeurs de transmissivité hydraulique estimées en concordance avec la texture du milieu poreux étudié.*

La carte des écarts-types de l'estimation du champ de transmissivité donne une idée de l'intervalle de confiance de notre résultat (Figure 13c). La plupart des incertitudes sur les transmissivités sont inférieures à 0.2 Log$_{10}$(T, in m^2 s^{-1}). L'inversion permet donc de reproduire correctement les comportements hydrauliques de l'aquifère (la racine carrée moyenne entre les données inversées et observée est de 3%) (Figure 11). Nous notons toutefois que la réponse hydrologique dans certains piézomètres (en particulier piézomètres # 3804 et # 3798) n'est pas toujours reproduite convenablement. À ces endroits, nous sommes probablement confrontés à un

problème de non-linéarité en raison d'un effet de mémoire des fluctuations des niveaux d'eau de la marée. Un tel effet cumulatif n'est pas reproduit par notre modèle. Dans les piézomètres # 3804 et # 3798, la différence entre les fluctuations hydrologiques calculées et observée est inférieure à 2 cm. C'est un excellent résultat, surtout si l'on considère que les variations de charge hydraulique sont de plus de 75 cm (dans le piézomètre # 3812 par exemple).

Dans la zone dominée par le forçage vertical lié aux apports latéraux de l'aquifère de la craie, le coefficient d'infiltration L de l'inversion est assez fort ($L \approx 10^{-1}$ jour^{-1} dans le voisinage des piézomètres # 3802 et # 3804). Ceci est cohérent avec le fait que, dans ce domaine, les tourbes de l'aquitard sont absentes ou très minces d'après les données des forages correspondants. D'autre part, les faibles valeurs du coefficient de forçage vertical sont compatibles avec les données des forages # 3522 et # 3808. Ces résultats sont confirmés par les observations hydrologiques illustrés par les fluctuations piézométriques enregistrées dans les deux aquifères (Figure 10c). L'influence du système d'endiguement est aussi responsable de la variabilité spatiale du forçage vertical. Des valeurs élevées de L ($\approx 10^{-1}$ jour^{-1}) ont été estimées dans des zones localisées soit à proximité du chenal soit à proximité des falaises. Nous notons que l'estimation de l'erreur-type pour le coefficient de forçage vertical varie entre 0.8 et 1.1 Log_{10} (L, en s^{-1}).

3.- Conclusion

Le fonctionnement hydrologique de l'aquifère semi-confiné de l'aquifère bi-couche du remplissage estuarien est caractérisé par son caractère hétérogène sous la double influence des marées et des apports latéraux de l'aquifère karstique de la craie. Nous avons développé une approche à la fois pour cartographier la distribution spatiale du champ de transmissivité et la distribution du coefficient de forçage vertical entre un aquifère semi-confiné et un aquifère non confiné. Les variations de marées enregistrées dans l'estuaire du fleuve sont utilisées en tant que conditions aux limites, afin de résoudre le problème hydraulique (équation de Darcy en régime transitoire). Le problème inverse consiste à reconstruire le champ de la transmissivité hétérogène à partir des observations de charge hydraulique en réponse aux fluctuations tidales. Ce problème inverse est non-linéaire et non-unique. On résout le problème inverse en générant un champ de transmissivité initial, krigé à partir des valeurs de quelques observations locales de la transmissivité. La transmissivité initiale est perturbée au niveau de 72 points pilotes pour reproduire les fluctuations de charge hydraulique. Ces points pilotes sont répartis uniformément dans l'aquifère selon une grille régulière. La perturbation optimale qui reproduit les variations piézométriques est déterminée grâce un algorithme de chaines de Markov Monte Carlo. Cette

approche a été testée avec succès sur un modèle synthétique et a été utilisée pour inverser le champ de transmissivité de l'aquifère semi-confiné des Marais du Hode.

III. La Tomographie hydraulique par des approches hydrogéophysiques

A.- L'utilisation des méthodes électriques pour la reconstruction de la perméabilité hydraulique des milieux poreux en domaines estuariens et côtiers

1.- Objectifs replacés dans leur contexte

Le potentiel spontané est la mesure passive en surface du sol ou en forage de la distribution du potentiel électrique naturellement crée par des circulations hydriques dans le sous sol. Pour cela, nous mesurons la différence de potentiel électrique entre deux électrodes impolarisables à l'aide d'un voltmètre de haute précision (1 µV) et de haute impédance d'entrée (> 10 Mohm). Ce phénomène d'électrofiltration génère un champ électrique en réponse à la circulation d'un fluide dans un milieu poreux ou fracturé. L'origine du couplage hydroélectrique est due à la présence d'un excès d'ions, lequel est entrainé par les mouvements de l'eau porale. Cet entrainement crée une densité du courant, laquelle est proportionnelle au gradient hydraulique dans le milieu poreux. La physique de ce phénomène est décrite dans un certain nombre de publications (par exemple, Jardani et al. [2006]) et ne sera pas répétée ici. Cette méthode a été utilisée récemment dans des applications en hydrogéologie pour déterminer la perméabilité du sol lors d'expériences de pompage (Rizzo et al. [2004]) ou pour déterminer les voies préférentielles des écoulements dans un milieu fracturé (Wishart et al. [2006]).

Dans ce cadre, une publication est en cours de finalisation dans laquelle nous avons exploré pour la première fois l'utilisation de la méthode du potentiel spontané pour suivre les fluctuations piézométriques générées par les oscillations tidales au sein des aquifères estuariens ou côtiers. La démarche proposée s'inscrit dans l'objectif de reconstruire la perméabilité hydraulique avec cette méthode non intrusive.

2.- Rappel du contexte de l'étude prospective

Le site d'essai est localisé dans les marais du Hode de l'estuaire de la Seine au Nord-Ouest de la France près du Havre (Figure 16). Ce site test est influencé par une marée forte avec une amplitude atteignant 8,4 m (Lesourd et al. [2001]). Le remplissage alluvial repose sur un anticlinal avec un substrat d'argiles du Gault à proximité du pied de la falaise puis les grès, sables et argiles du Crétacé inférieur sur la plaine alluviale restante (figure 3). Dans la basse vallée de la Seine, les formations fluviatiles sont composées de trois séquences (Figure 3, Lesourd et al. [2001]), à la base, de graves et sables graveleux perméables, surmontés par un

niveau plus ou moins épais et, plus ou moins discontinu, de tourbes et matériaux fins très peu perméables puis, par une formation perméable de sables et silts qui est toutefois relativement moins transmissive que les formations graveleuses de la base de la série sédimentaire.
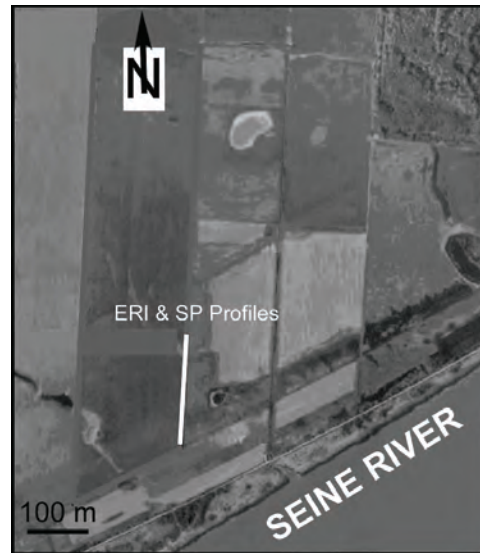

Figure 15. *Photographie aérienne montrant le site d'étude localisé dans l'estuaire de la Seine soumis à un régime mégatidal : positionnement du profil d'imagerie de résistivité électrique (ERI, Electrical Resistivity Imaging) et de potentiel spontané (SP).*

3. Principaux résultats

L'imagerie de résistivité électrique (IRE) a été réalisée le long d'un profil qui est perpendiculaire à la Seine (voir la position de la figure 16). Pour cette étude, nous avons utilisé Wenner-α comme quadripôle avec 48 électrodes et 4 m d'écartement entre les électrodes. Les données ont été inversées en utilisant le logiciel Res2Dinv (Loke et Barker. [1996]). L' IRE a permis de différencier les dépôts fins et grossiers caractérisés par la résistivité électrique ~ 30 et ~ 6 Ohm.m, respectivement (figure 17). Ces résultats sont cohérents avec les données lithologiques des deux forages dans la zone d'étude où les épaisseurs de sédiments fins et grossiers graviers sont, respectivement de 21,5 m et 10 m (figure 18). La résistivité électrique de chaque formation reflète la conductivité de l'eau interstitielle mesurée dans les deux piézomètres : σ_f=4000 μS cm^{-1} dans l'aquifère libre et σ_f=14000 μS cm^{-1} dans l'aquifère semi-confiné.
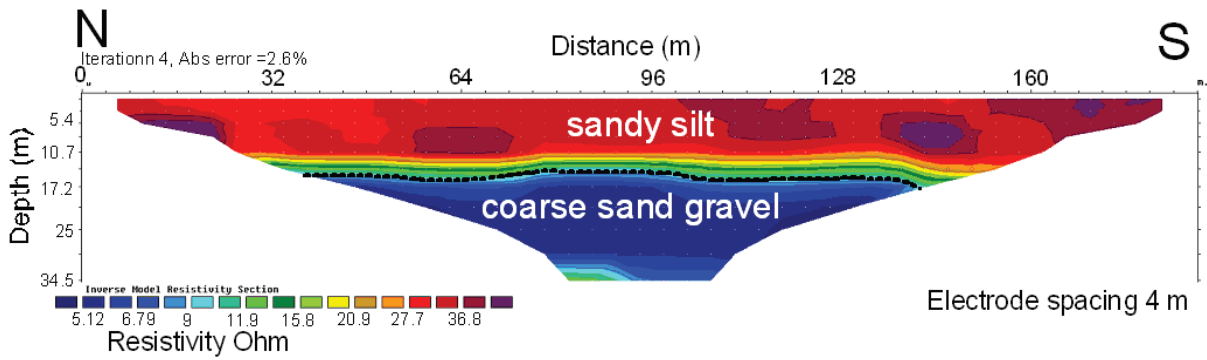
Figure 16. *Profil de résistivité électrique permet d'identifier : (1) la formation supérieure de silts et sables fins avec des résistivités électriques ~ 30 ohm.m^{-1} et (2) la formation de base des sables grossiers, graviers et cailloutis avec des résistivités électriques ~ 6 ohm.m^{-1}.*
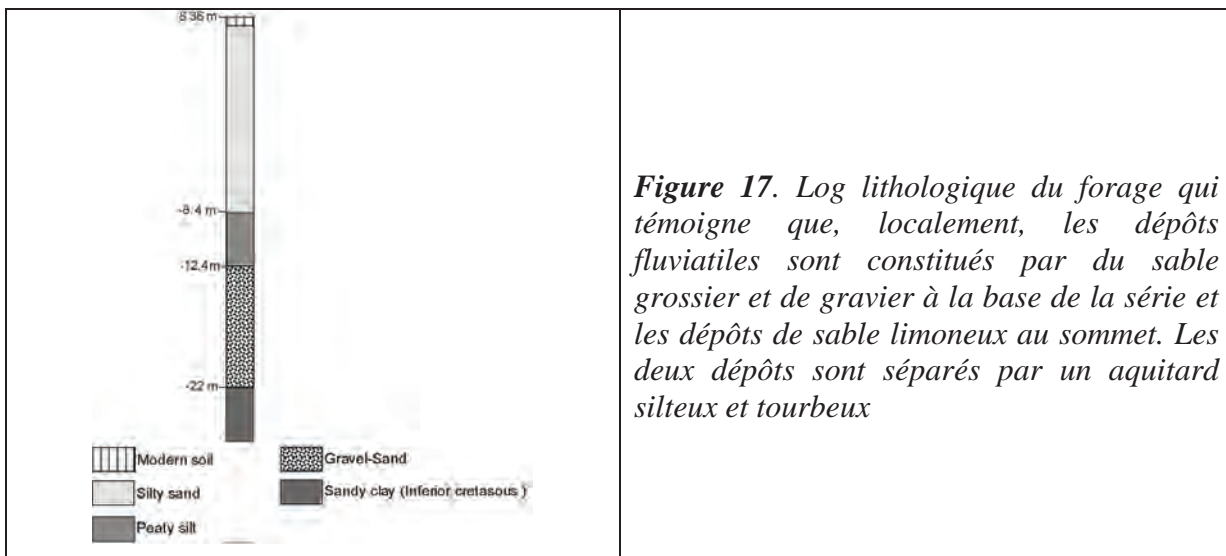
Figure 17. *Log lithologique du forage qui témoigne que, localement, les dépôts fluviatiles sont constitués par du sable grossier et de gravier à la base de la série et les dépôts de sable limoneux au sommet. Les deux dépôts sont séparés par un aquitard silteux et tourbeux*

Les enregistrements des données de potentiel spontané ont été effectués pour suivre les fluctuations de charge hydraulique résultant des oscillations naturelles de marée. Les mesures ont été effectuées sur une période de 55 heures avec un pas de temps d'échantillonnage de 5 minutes. Une série de neuf électrodes non polarisables a été installée à la surface du sol avec un écartement de 10 m entre les électrodes. Ces électrodes sont reliées à un enregistreur multi-canal DC-voltmètre. Comme les mesures de potentiel spontané peuvent être contaminées par les signaux électriques anthropiques ou naturels, la série temporelle de potentiel spontané a été analysée en utilisant une décomposition en ondelettes multi-résolution pour mieux extraire le signal lié à l'influence des marées. L'analyse multi-résolution a été effectuée en utilisant les ondelettes de Daubechies en extrayant huit composantes couvrant la période d'enregistrement de PS (Figure 18).
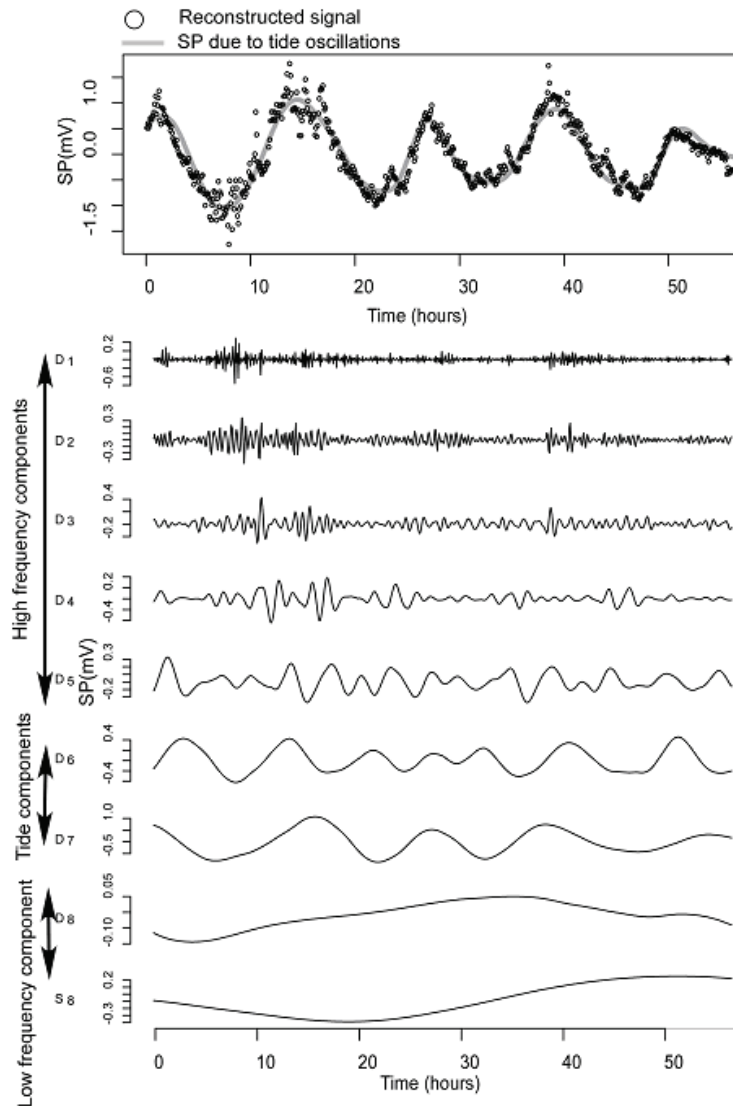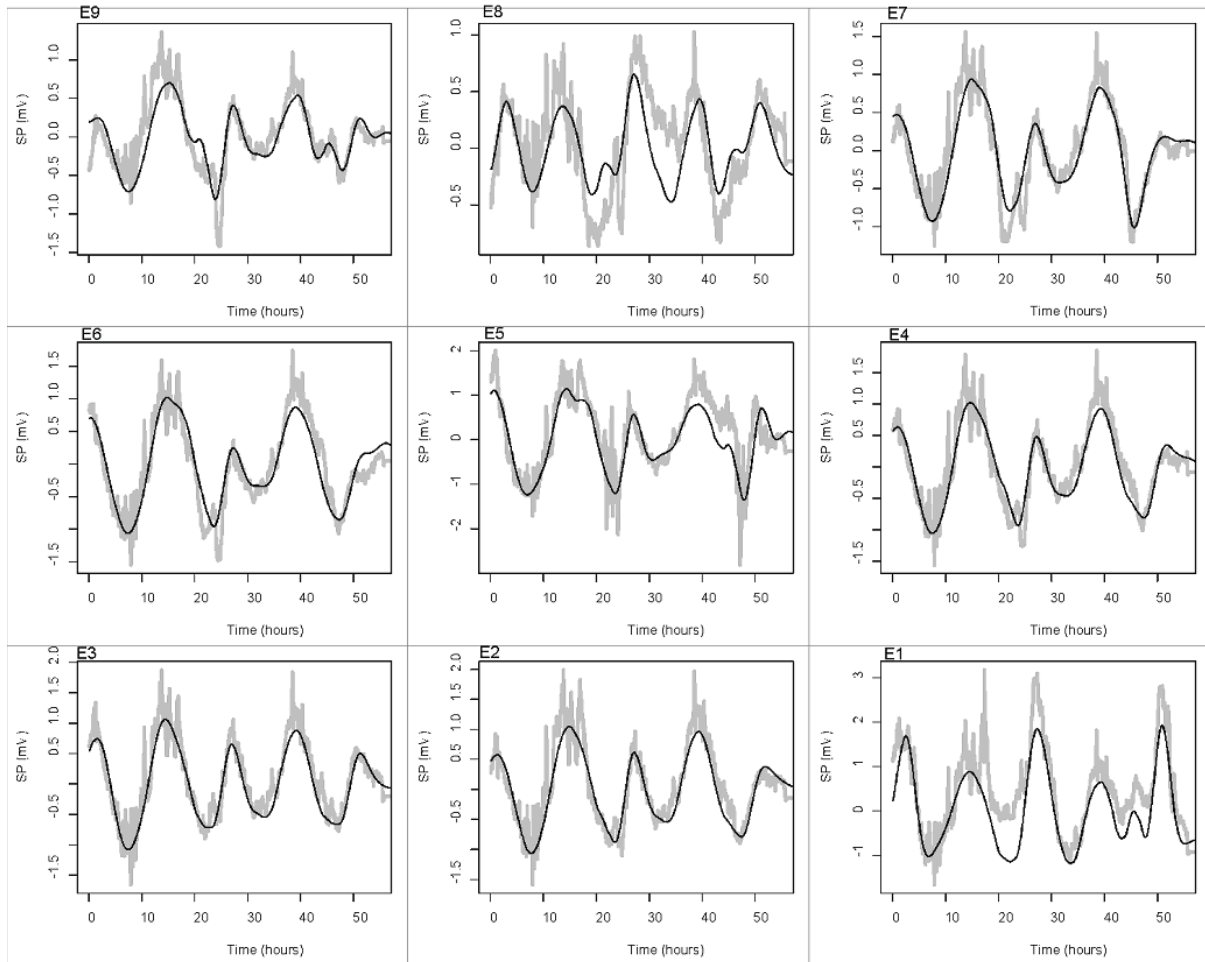
Figure 18*. Analyse multi-résolution de la série temporelle SP recueillies avec l'électrode E3. Le signal SP mesuré peut être décomposé en une somme de huit composantes. La première classe des composantes décrit les phénomènes de haute fréquence principalement liés aux sources de bruit (D1, D2, D3 et D4). Les composantes suivantes notées D6 et D7 représentent les fluctuations de potentiel spontané résultant des oscillations de charge hydraulique des aquifères avec une période de l'ordre de 12 heures qui correspond aux signaux de la marée. Puis, les basses fréquences inférieures à D7 permettent de reconstituer l'influence diurne de la température sur le signal de potentiel spontané.*

Les composantes obtenues à partir de la décomposition multi-résolution peuvent être classées en deux catégories. Les premières composantes qui sont liées au bruit et qui sont caractérisées par des signatures de hautes fréquences (D1, D2, D3, D4 et D5). Dans la seconde catégorie, les composantes représentent le comportement réel du signal de potentiel spontané associé au contexte hydrogéophysique (Figure 18) : (1) la somme des composantes D6 et D7 qui permet de reconstruire le comportement sinusoïdal de la charge hydraulique de l'aquifère avec une période de 12 heures, typique de l'influence des marées et (2) les composantes de basse

fréquence (inférieure à D7) qui permettent d'extraire du signal SP l'influence des changements de température à l'échelle de la journée (Figure 19).

Cette signature électrique est donc associée à des oscillations périodiques de la position de l'interface entre les zones vadose et saturée, provoquées par les variations de marée. Un modèle numérique décrivant la physique comprise dans ce processus est discuté **en Annexe 3**

Figure 19. *Séries temporelles du signal SP, enregistrées sur les neuf électrodes avec les signaux mesurés et les signaux filtrés (D6 et D7) à partir de l'analyse multi résolution pour extraire les oscillations harmoniques des variations de marée dans les aquifères.*

4. Modélisation et conclusions

Les fluctuations de charge hydraulique liées aux oscillations de marées sont exploitables en tant que sources d'information pour évaluer la conductivité hydraulique, des aquifères de subsurface. Dans cette étude, nous démontrons que, en l'absence de données piézométriques, la méthode non-intrusive du potentiel spontané peut être une alternative pour estimer les paramètres hydrauliques (la perméabilité hydraulique et le coefficient d'emmagasinement) des milieux hétérogènes afin d'établir un modèle numérique d'écoulement dans un milieu saturé et non saturé.
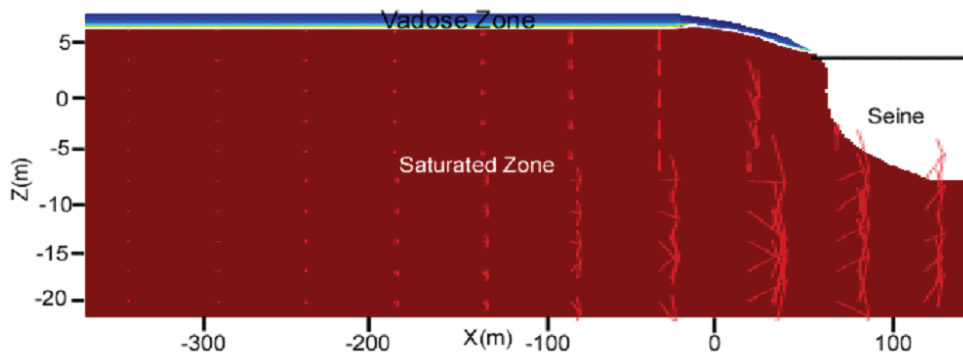
Figure20. *La figure présente l'état de saturation du milieu durant la marée basse obtenu par la reconstruction des paramètres hydraulique avec l'inversion des mesures de potentiel spontané*

B.- L'utilisation de la méthode électromagnétique de basse fréquence

La méthode électromagnétique de basse fréquence EM34 s'agit d'un outil géophysique composé de deux boucles (émetteur et récepteur) qui mobilise donc deux personnes pendant l'acquisition des profils de mesures. L'espacement entre les deux boucles peut être fixé à 10, 20 et 40 m en fonction de la profondeur de pénétration et de la résolution désirée. De ce fait, les fréquences correspondantes sont de 6400, 1600 et 400 Hz. Il est possible d'utiliser la méthode EM34 selon deux dispositifs : HDM Horizontal Dipole Mode ou Vertical Dipole mode (VDM). Lorsque les deux boucles sont verticales, la contribution de la couche la plus proche de surface est très importante et la contribution de la couche située à grande profondeur devient importante lorsque les boucles sont horizontales.

L'avantage de la méthode EM34 est qu'elle permet d'obtenir une mesure rapide de la conductivité du milieu. Il est possible de couvrir facilement une grande zone d'étude afin de détecter les variations de conductivité apparente du sous-sol en profondeur. Dans ce projet, nous avons cherché à identifier les origines des masses d'eau dans le sous-sol à partir de leur conductivité électrique mesurée. La méthode EM 34 a été utilisée avec les deux modes vertical et horizontal avec une distance de 10 m pour les deux profondeurs d'exploration 15m et 7.5m dans des conditions climatiques hivernales de 2012 et avec une saturation totale du milieu (la zone était totalement inondée durant la campagne de mesure). Les distributions spatiales de la conductivité apparente enregistrées avec les deux modes : vertical (Figure22. 1) et horizontal (Figure22. 2) ont permis d'identifier les zones dominées par une diffusion saline provenant du chenal de la Seine. Elles sont caractérisées par une forte conductivité électrique.

Par contre, les zones proches des falaises de craie, soumises aux apports dominants de l'aquifère karstique, la conductivité électrique est faible de l'ordre de 15 mS/m du fait de la faible minéralisation des eaux de la craie par rapport aux eaux de la seine.
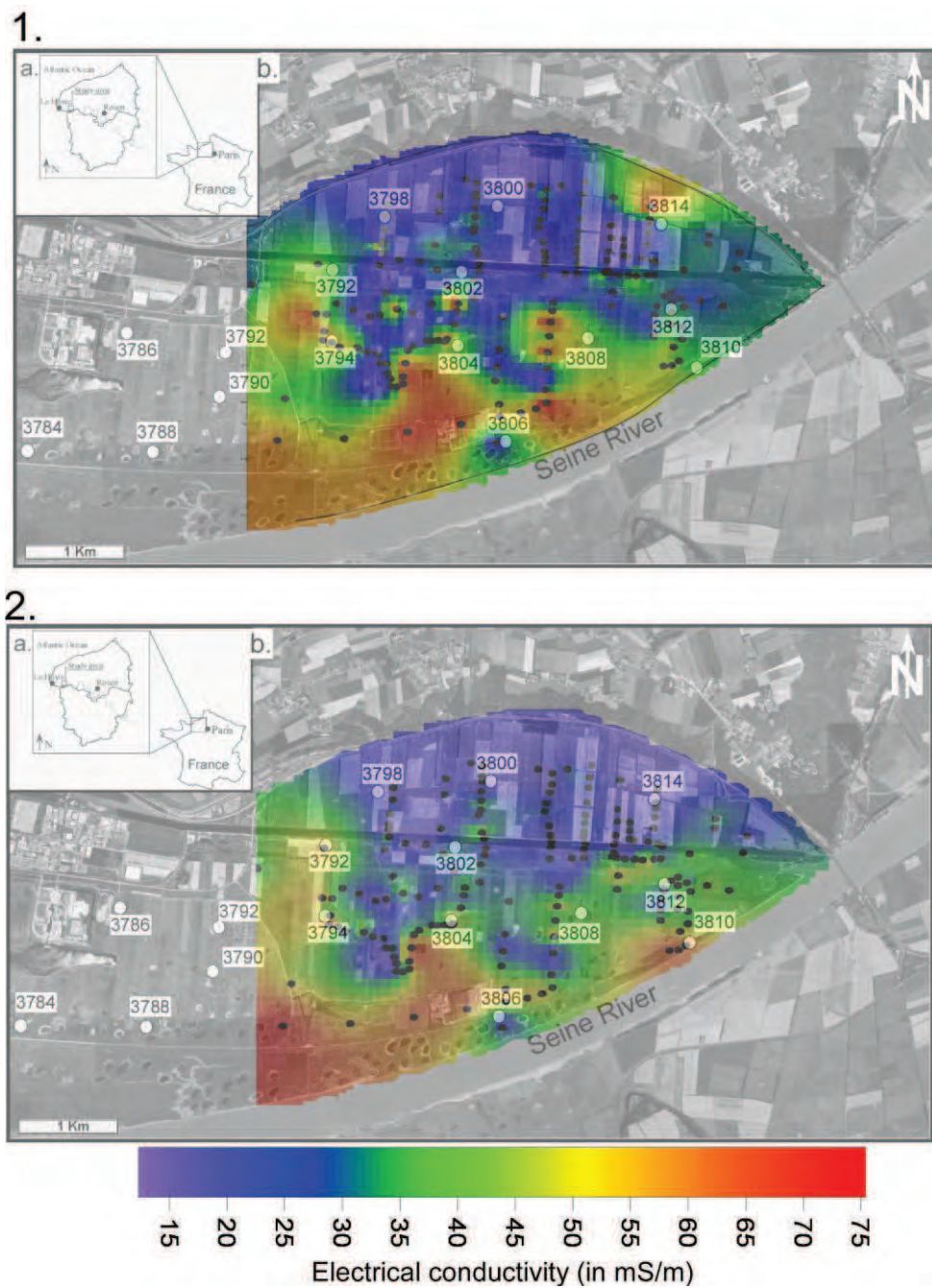
Figure 21. Les distributions spatiales de la conductivité apparente enregistrées avec les deux modes : vertical (Figure 22. 1) et horizontal (Figure22. 2).

Ces résultats de la conductivité électriques sont cohérents avec les analyses chimiques des eaux des aquifères semi confiné et non confiné prélevées dans les piézomètres du site d'étude qui

présentent une gamme de variation géochimique entre un pôle chloruré sodique et sulfaté lié à l'influence des apports marins et un pôle hydrogénocarbonaté calcique et magnésien, caractéristique des apports de l'aquifère de la craie. La minéralisation totale peut être abordée par les mesures de conductivité électrique qui témoignent qu'il s'agit généralement d'eaux fortement minéralisées (1000 à 5000 µS/cm contre 450 à 700 µS/cm pour les eaux de la craie) dont les teneurs en sels sont toutefois très largement inférieures à celles atteintes dans le chenal de la Seine. Ce qui explique l'augmentation de la conductivité électrique du milieu poreux autour du chenal de la seine.

II. Approche hydrochimique

Nous dédions cette section à la caractérisation des paramètres physico-chimiques de masse d'eau prélevée durant trois compagnes réalisées en janvier et Juin 2008 pour l'objectif de discerner les origines des masses d'eau de les deux aquifères. Durant ces trois compagnes nous avons déterminé les paramètres et les concentrations des éléments chimiques suivants : pH, Oxygène dissous, salinité, Matière en suspension, nitrates, Ammonium, Hydrogénocarbonates calcium, potassium, magnésium, zinc, phosphore, PCB, HAP et Tritium. Mais nous focalisons notre discussion seulement sur les éléments marqueurs des eaux de la seine, marine et karstique.

1. Méthodologie d'analyse et de traitement des données

1.1. Analyses géochimiques

Diagramme de Piper

Le diagramme de Piper permet une représentation des anions et des cations sur deux triangles spécifiques dont les côtés témoignent des teneurs relatives en chacun des ions majeurs par rapport au total de ces ions (cations pour le triangle de gauche et anions pour le triangle de droite). La position relative d'un résultat analytique sur chacun de ces deux triangles permet de préciser en premier lieu la dominance cationique et anionique.

A ces deux triangles, est associé un losange sur lequel est reportée l'intersection des deux lignes issues des points identifiés sur chaque triangle (Figure 22). Ce point d'intersection représente l'analyse globale de l'échantillon, sa position relative permet de préciser le faciès de l'eau concernée.
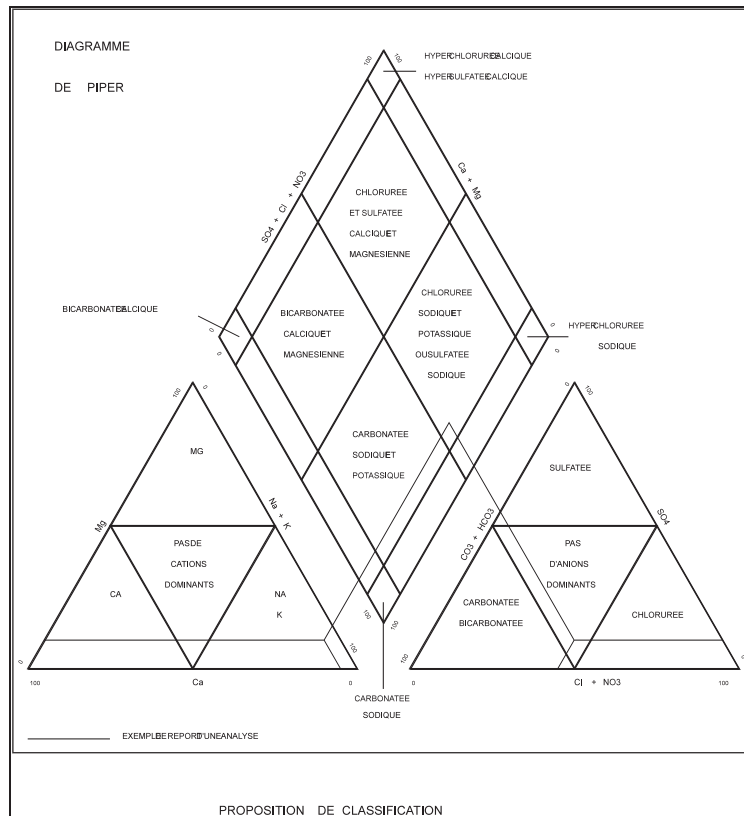
Figure 22. *Exemple de résultats analytiques sur le diagramme de Piper*

- Sommet du losange = faciès sulfaté / chloruré, calcique / magnésien
- Base du losange = faciès bicarbonaté / sodique
- Pointe droite du losange = faciès sulfaté / chloruré, sodique
- Pointe gauche du losange = faciès bicarbonaté, calcique / magnésien.

Diagramme de Schöller-Berkaloff

Le diagramme de Schöller-Berkaloff est une représentation graphique semi-logarithmique. Sur l'axe des abscisses sont représentés les différents ions. Pour chacun de ces ions (ou des groupements Na^+ + K^+ et CO_3^- + HCO_3^-) la teneur réelle en mg/l est reportée sur l'axe des ordonnées. Les points obtenus sont reliés par des droites. L'allure du graphique obtenu permet de visualiser le faciès de l'eau minérale concernée.
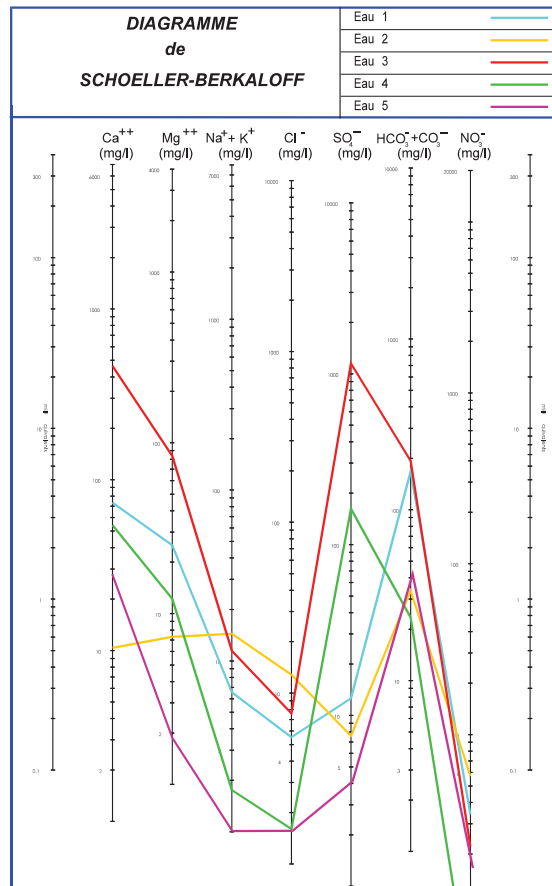
Figure 23*: Exemple de diagramme de Schöller-Berkaloff*

Comparaison des deux types de diagrammes

Le problème du diagramme de Piper vient du fait que les analyses sont reportées sous forme de pourcentage, ainsi l'effet de dilution (variation de la concentration absolue) n'est pas immédiatement perceptible. Il convient d'être prudent dans la comparaison des qualités physico-chimiques de deux échantillons distincts.

L'avantage de Piper à contrario est qu'il permet de représenter sur un même graphique de nombreuses analyses autorisant des regroupements par famille présentant des faciès similaires.

Le diagramme de Schöller-Berkaloff permet une lecture directe des concentrations en mg/l des différents ions majeurs, sans conversion en % relatif (cf. diagramme de Piper). En outre, les deux axes des ordonnées situés à droite et à gauche du graphique permettent une conversion directe en milliéquivalent (rapport de la concentration en mg/l d'un ion donné à son équivalent chimique A/N avec A : masse atomique et N : valence de l'ion). Ces deux modes de représentation facilitent à la fois la construction des graphiques et la lecture des données.

Si les concentrations sont identiques alors les droites sont superposées et dans le cas contraire il y a un décalage relatif des droites les unes au dessus des autres.

Un inconvénient de ce diagramme réside dans le fait que la comparaison de deux eaux peut amener à considérer que les faciès physico-chimiques sont similaires alors que les écarts des concentrations relatives en un ou plusieurs éléments sont significatifs.

Ces deux types de représentations des analyses d'eau sont donc complémentaires quant à l'interprétation des faciès physico-chimiques que l'on peut en déduire.

III. Résultats des analyses géochimiques -Campagne de Janvier 2008

L'ensemble des données géochimiques effectuées sont représentées sur le diagramme de Piper (Figure 24). Un code couleur permet de distinguer les piézomètres longs (vert), les piézomètres moyens (orange) et ce que nous appellerons les références (bleu). Les références sont les données géochimiques des échantillons d'eau du Canal de Tancarville (CT) et du Grand Canal du Havre (GCH).
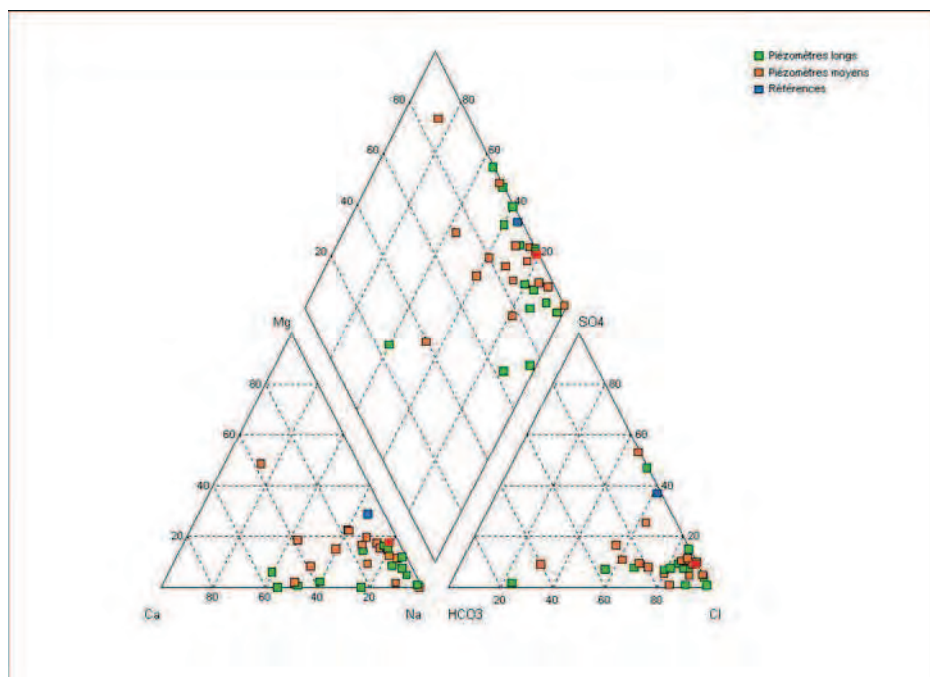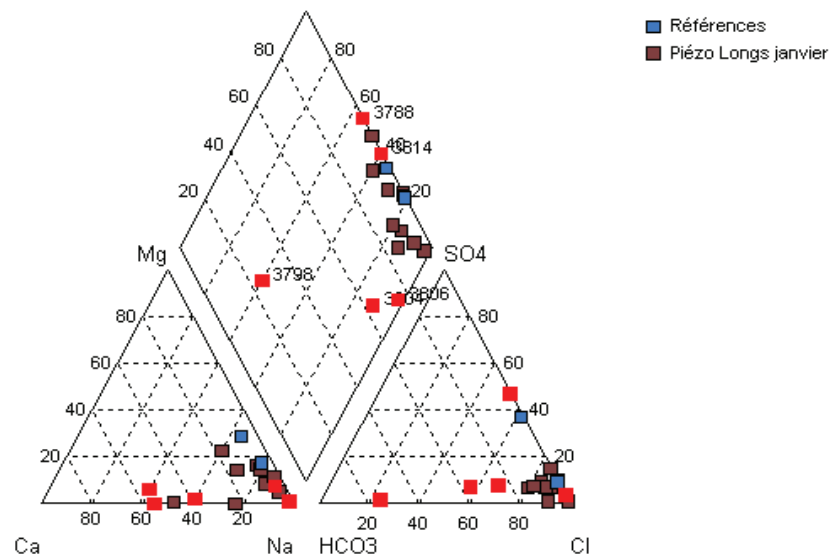

Figure 24: Diagramme de Piper des échantillons d'eau prélevés dans l'ensemble des piézomètres ainsi que dans le Canal de Tancarville et le Grand Canal du Havre durant la campagne de janvier 2008.

Afin de mieux appréhender les résultats des analyses géochimiques, nous avons réalisé deux diagrammes de Piper supplémentaires afin de distinguer les faciès des piézomètres longs et ceux des piézomètres moyens.

Le diagramme de Piper de piézomètres longs (Figure 25) indique que 5 piézomètres se distinguent d'un point de vue géochimique de l'ensemble des autres.

En effet, le triangle des anions indique une dominance des chlorures à l'exception de 3798 (dominance des bicarbonates) et 3814 (limites des dominances). Le triangle des cations indique une dominance du sodium/potassium à l'exception de 3798 et 3814 (dominance du calcium). Le losange des faciès indique un faciès sulfaté/chloruré sodique dont 3806 et 3804 sont légèrement excentrés, mais 3798 présente quant à lui un faciès bicarbonaté calcique/magnésien.

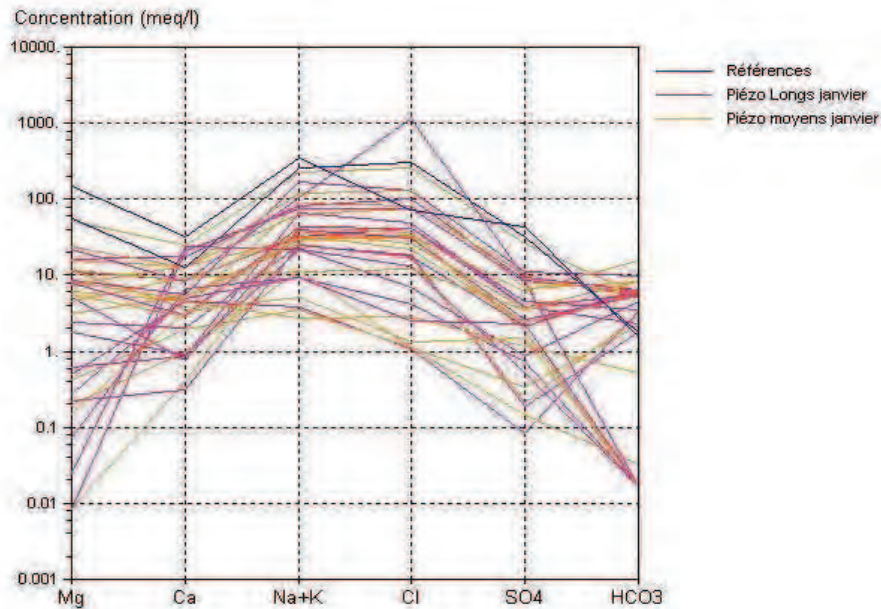**Figure 25**: Diagramme de Piper des échantillons d'eau prélevés dans les piézomètres longs.

Le diagramme de Piper de piézomètres moyens (Figure 26) indique que 4 piézomètres se distinguent d'un point de vue géochimique de l'ensemble des autres.

En effet, le triangle des anions indique une dominance des chlorures à l'exception de 3799 (dominance des bicarbonates) et 3815 (dominance des sulfates). Le triangle des cations indique une dominance du sodium/potassium à l'exception de 3805 (dominance du magnésium). Le losange des faciès indique un faciès sulfaté/chloruré sodique, mais 3799 présente un faciès bicarbonaté sodique et 3811-3815-3805 présentent un faciès sulfaté/chloruré magnésien/calcique.
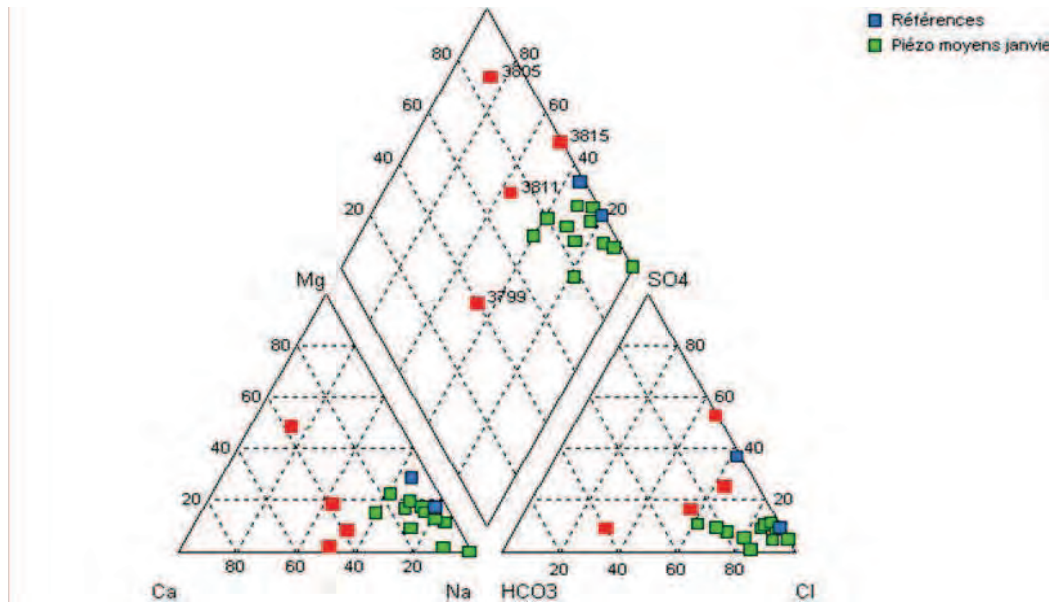
Figure 26: Diagramme de Piper des échantillons d'eau prélevés dans les piézomètres moyens.

L'ensemble des données géochimiques effectuées sont également été représentées sur le diagramme de Schöller-Berkaloff (Figure 27). Un code couleur permet de distinguer les piézomètres longs (rose), les piézomètres moyens (orange) et ce que nous appellerons les références (bleu). Les références sont les données géochimiques des échantillons d'eau du Canal de Tancarville (CT) et du Grand Canal du Havre (GCH).

Figure 27. Diagramme de Schöller-Berkaloff des échantillons d'eau prélevés dans l'ensemble des piézomètres accompagnés des références (CT et GCH)

Afin de mieux appréhender les résultats des analyses géochimiques, nous avons réalisé deux diagrammes de Schöller-Berkaloff supplémentaires afin de distinguer les familles de concentration des piézomètres longs et ceux des piézomètres moyens.

Le diagramme de Schöller-Berkaloff des piézomètres longs nous permet de distinguer trois types de « familles ». Les piézomètres présentant une ligne brisée différente de la majorité des piézomètres sont mis en surbrillance rouge, ce sont les piézomètres 3788, 3796, 3800 et 3814 pour le diagramme du haut et le piézomètre 3810 pour le diagramme du bas (Figure 28).
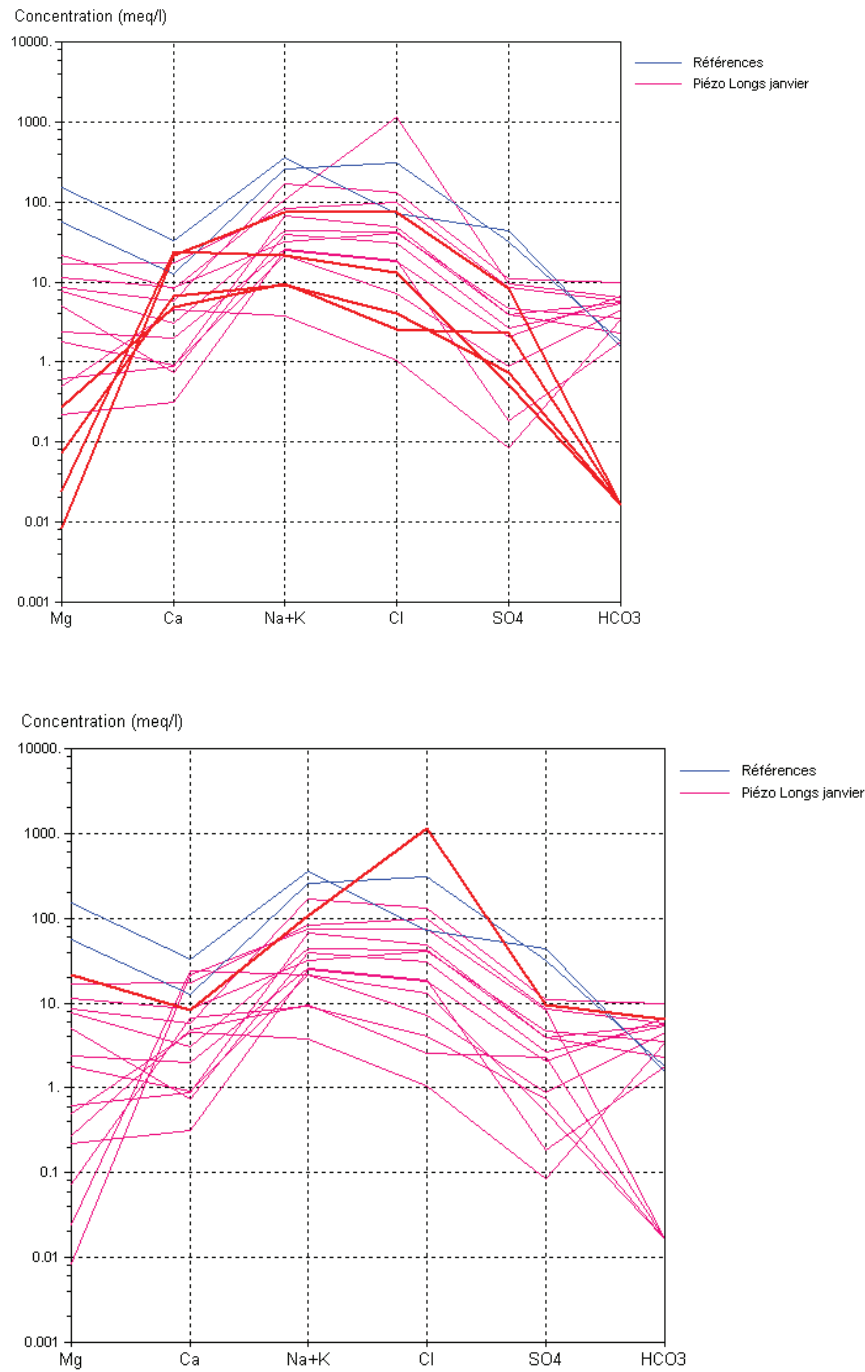
Figure 28. Diagrammes de Schöller-Berkaloff des échantillons d'eau des piézomètres longs accompagnés des références (CT et GCH).

Le diagramme de Schöller-Berkaloff des piézomètres moyens nous permet de distinguer deux types de « familles ». Les piézomètres présentant une ligne brisée différente de la majorité des piézomètres sont mis en surbrillance rouge, ce sont les spz 3797, 3801 et 3815 (Figure 29).

Figure 29. Diagramme de Schöller-Berkaloff des échantillons d'eau des piézomètres moyens accompagnés des références (CT et GCH).

Campagne de Juin
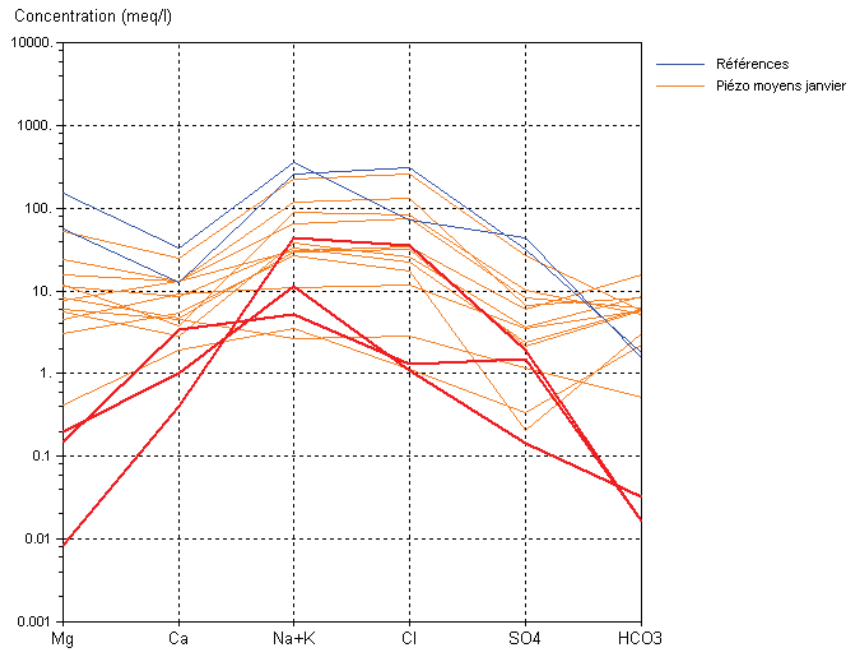

L'ensemble des données géochimiques effectuées sont représentées sur le diagramme de Piper (Figure 30). Un code couleur permet de distinguer les piézomètres longs (vert), les piézomètres moyens (rose), les piézomètres courts (rouge) et ce que nous appellerons les références (bleu). Les références sont les données géochimiques des échantillons d'eau du Canal de Tancarville (CT) et du Grand Canal du Havre (GCH).
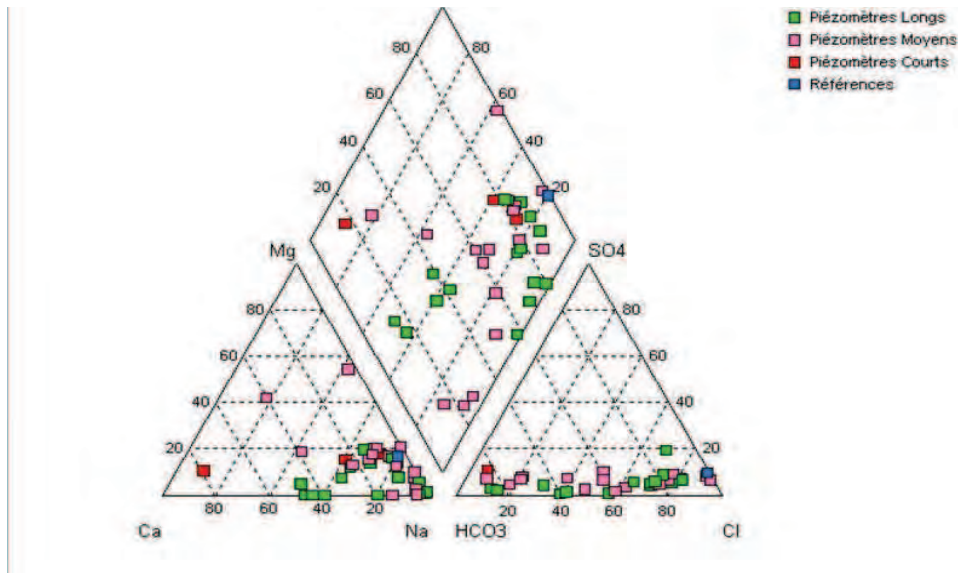
Figure 30. Diagramme de Piper des échantillons d'eau prélevés dans l'ensemble des piézomètres accompagnés des références (CT et GCH) lors de la campagne de juin 2008.

Le diagramme de Piper de piézomètres longs (Figure 31) indique que 5 piézomètres se distinguent d'un point de vue géochimique de l'ensemble des autres.

En effet, le triangle des anions indique une dominance des chlorures à l'exception de 3798-3814-3788-3804-3800 (dominance des bicarbonates). Le triangle des cations indique une dominance du sodium/potassium. Le losange des faciès indique un faciès sulfaté/chloruré sodique, mais 3798-3814-3788-3804-3800 présentent un faciès bicarbonaté sodique.

Figure 31. Diagramme de Piper des échantillons d'eau prélevés dans les piézomètres longs accompagnés des références (CT et GCH).
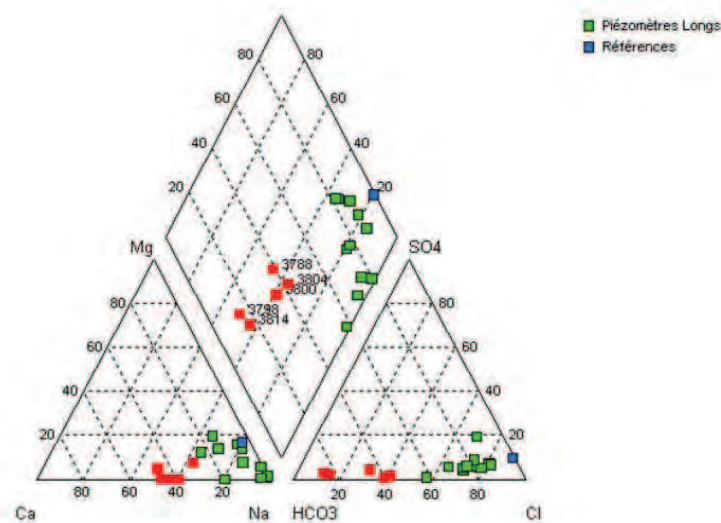
Le diagramme de Piper de piézomètres moyens (Figure 32) indique que 6 piézomètres se distinguent d'un point de vue géochimique de l'ensemble des autres.

En effet, le triangle des anions indique une dominance des chlorures à l'exception de 3815-3799-3811-3805-3801 (dominance des bicarbonates). Le triangle des cations indique une dominance du sodium/potassium à l'exception de 3809 (dominance du magnésium) et 3805-3811 (pas de dominance). Le losange des faciès indique un faciès sulfaté/chloruré sodique, mais 3815-3799-3801 présentent un faciès bicarbonaté sodique; 3805-3811 présentent un faciès bicarbonaté calcique/magnésie et 3809 un faciès sulfaté/chloruré calcique/magnésien.

Figure 32. Diagramme de Piper des échantillons d'eau prélevés dans les piézomètres moyens accompagnés des références (CT et GCH)

Le diagramme de Piper de piézomètres courts (Figure 33) indique que 1 piézomètre se distingue d'un point de vue géochimique de l'ensemble des autres.
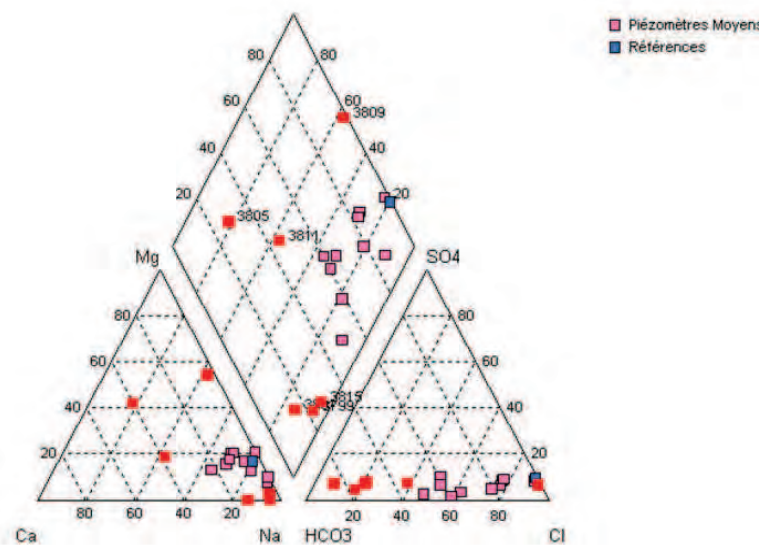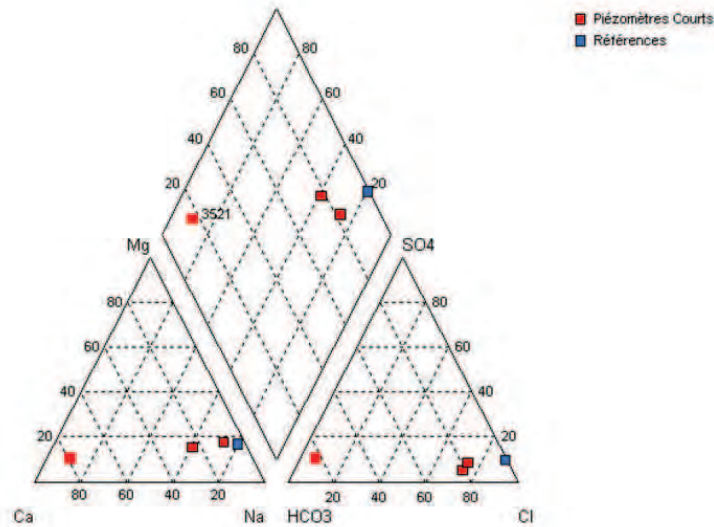
En effet, le triangle des anions indique une dominance des chlorures à l'exception de 3521 (dominance des bicarbonates). Le triangle des cations indique une dominance du sodium/potassium à l'exception de 3521 (dominance du calcium). Le losange des faciès indique un faciès sulfaté/chloruré sodique, mais 3521 présente un faciès bicarbonaté calcique/magnésien.

Figure 33. Diagramme de Piper des échantillons d'eau prélevés dans les piézomètres courts accompagnés des références (CT et GCH).

L'ensemble des données géochimiques effectuées sont également été représentées sur le diagramme de Schöller-Berkaloff (Figure 34). Un code couleur permet de distinguer les piézomètres longs (bleu), les piézomètres moyens (vert), les piézomètres courts (mauve) et ce que nous appellerons les références (noir). Les références sont les données géochimiques des échantillons d'eau du Canal de Tancarville (CT) et du Grand Canal du Havre (GCH).
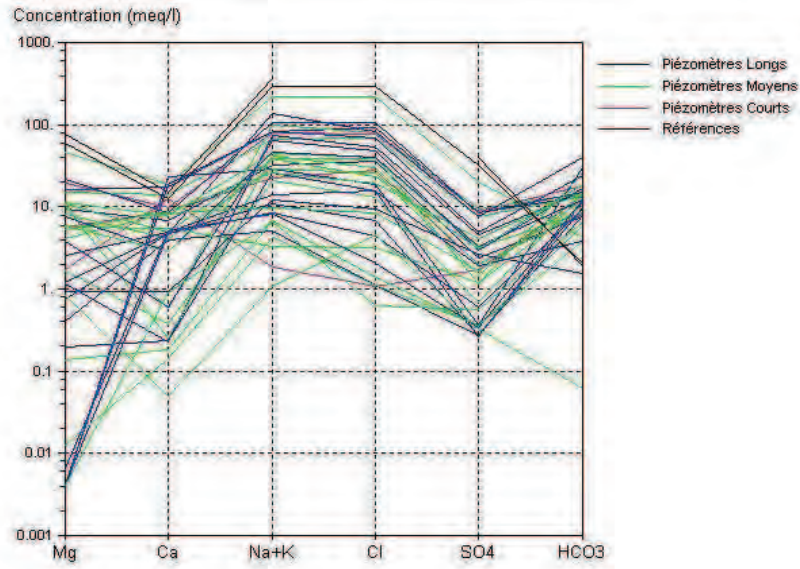
Figure 34. Diagramme de Schöller-Berkaloff des échantillons d'eau prélevés dans l'ensemble des piézomètres accompagnés des références (CT et GCH).

Afin de mieux appréhender les résultats des analyses géochimiques, nous avons réalisé trois diagrammes de Schöller-Berkaloff supplémentaires afin de distinguer les familles de concentration des piézomètres longs, des piézomètres moyens et des piézomètres courts.

Le diagramme de Schöller-Berkaloff des piézomètres longs nous permet de distinguer deux types de « familles ». Les piézomètres présentant une ligne brisée différente de la majorité des

piézomètres sont mis en surbrillance rouge, ce sont les piézomètres 3788, 3796, 3800 et 3814 (Figure 35).

Figure 35. Diagramme de Schöller-Berkaloff des échantillons d'eau des piézomètres longs accompagnés des références (CT et GCH).

Le diagramme de Schöller-Berkaloff des piézomètres courts nous permet de distinguer deux types de « familles ». Le piézomètre présentant une ligne brisée différente des deux autres

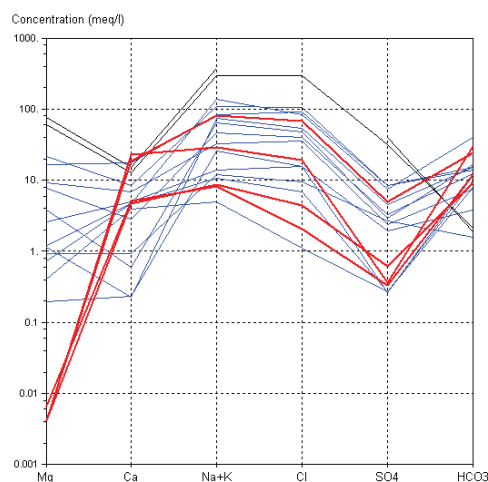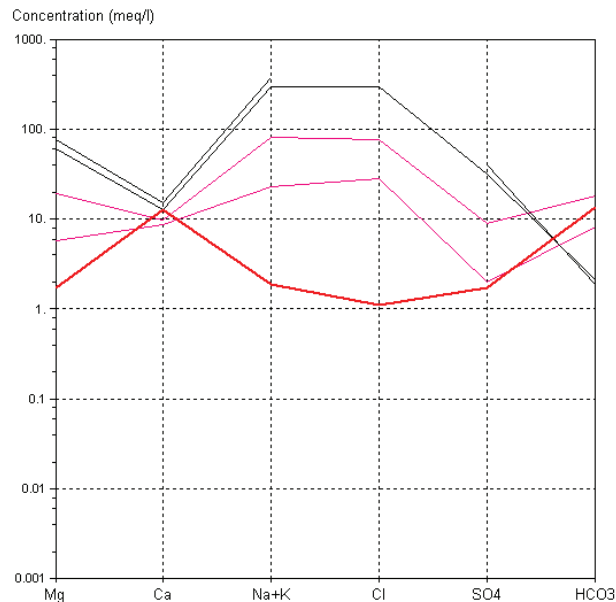piézomètres est mis en surbrillance rouge, c'est le piézomètre 3521 (Figure 36).

Figure 36. Diagramme de Schöller-Berkaloff des échantillons d'eau prélevés dans les piézomètres courts accompagnés des références (CT et GCH).

Durant cette même campagne de Juin 2008, des analyses de l'activité du Tritium (H_3) ont été réalisées. Afin de faciliter leur interprétation, les données de l'activité en Tritium exprimées en Becquerel ont été intégrées sur une carte de localisation des piézomètres (Figure 37). Un code couleur permet de distinguer les piézomètres longs (bleu), les piézomètres moyens (rose) et les piézomètres courts (vert).

Les données de tritium s'interprètent de la façon suivante :
-plus la concentration est élevée plus les eaux sont « vieilles »
-plus la concentration est faible plus les eaux sont « jeunes ».

De manière globale, les piézomètres longs donc les graves de fond devraient avoir les concentrations les plus faibles par apport d'eaux « jeunes » de la nappe. Ainsi, les piézomètres moyens auraient des concentrations plus grandes dues aux accidents de centrale radioactive. Ces

valeurs seraient encore importantes d'une part parce que ces eaux sont peu remaniées et d'autre part parce que la demi-vie du tritium étant de 12.34 ans sa dégradation est lente.

Ceci se vérifie pour 9 points de sondage sur 16. Il y a donc 5 points de sondage pour lequel cela ne se vérifie pas. Ce sont les paires de piézomètres : 3804/3805 ; 3796/3797 ; 3792/3793 ; 3788/3789 ; 3784/3785.
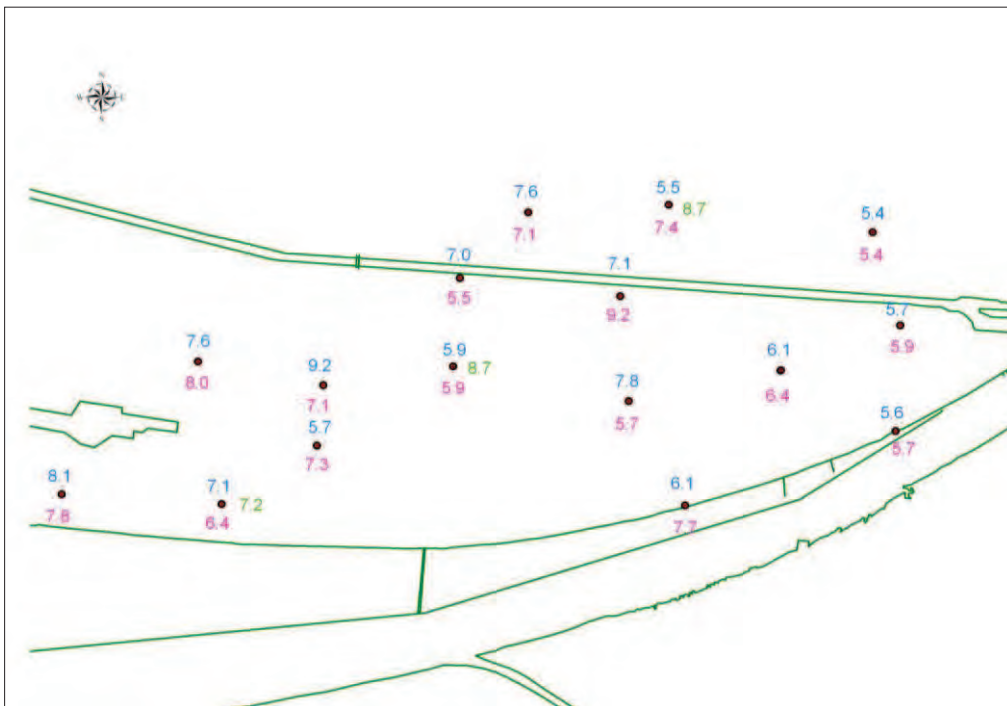
Figure 37 . Carte de l'activité du Tritium en Becquerel selon la localisation des piézomètres.

VI. Les données géochimiques pour la construction d'un modèle de mélange des apports dans les aquifères semi confiné et non confiné de la zone d'étude

La géochimie des aquifères semi confiné et non confiné du remplissage estuarien est complexe puisqu'elle résulte du mélange entre les apports latéraux d'eaux souterraines de la nappe de la craie, les apports marins et estuariens qui transitent latéralement dans les milieux poreux et les apports de l'inondation contrôlés par les gestionnaires des zones humides.

Les données utilisées dans cette approche proviennent :

- pour l'aquifère de la craie, des données qualitométriques de la base de données ADES ;
- pour la géochimie des eaux de l'estuaire, des données qualitométriques de la base de données Seine aval à partir de relevés Tancarville, Berville, Honfleur, réalisés dans le gradient de salinité de l'estuaire ;
- pour les eaux des aquifères semi confiné et non confiné, des données qualitométriques collectées par le GPMH dans les piézomètres profonds et moyens ;
- pour les eaux pluviales, des données qualitométriques réalisées dans le cadre d'un suivi réalisé en forêt de Brotonne (communication DREAL) ;

Les eaux de l'aquifère de la craie étant bicarbonatées calciques et magnésiennes et faiblement chlorurées, nous nous intéressons d'abord au statut du Calcium dans les apports estuariens.

Les eaux du gradient de salinité de l'estuaire témoignent d'un niveau quasi constant des concentrations de Calcium dans le mélange (Figure 38).
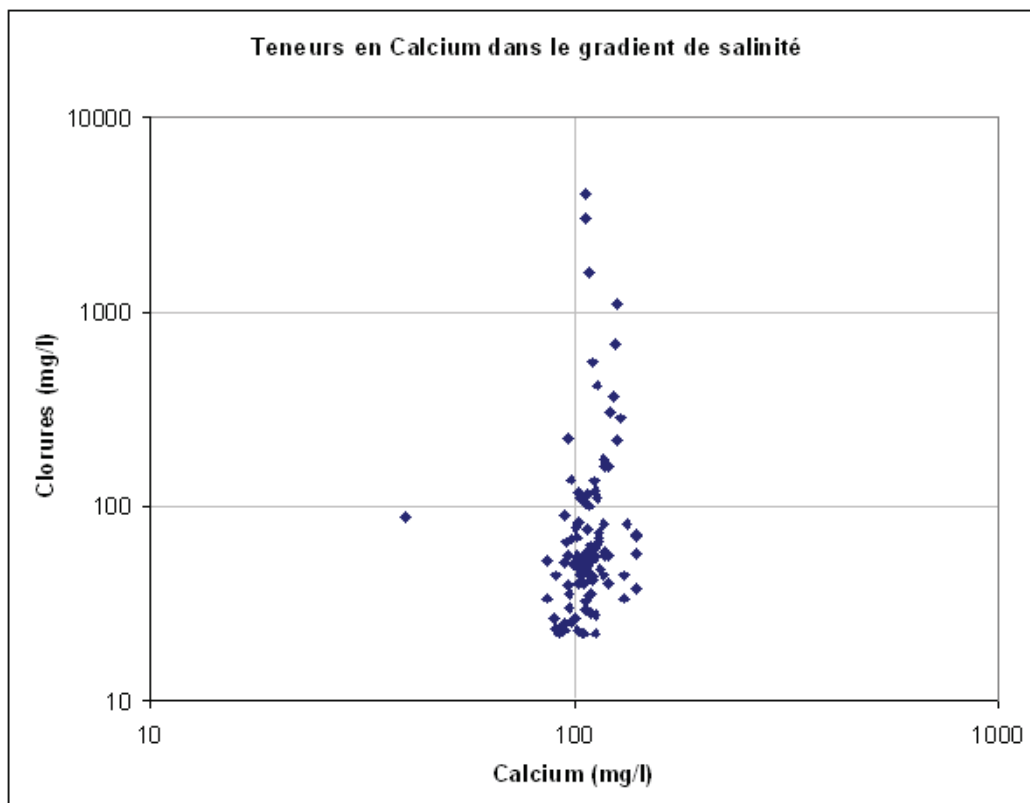
Figure 38. Teneurs en calcium dans le gradient de salinité des eaux du chenal de Seine entre Tancarville et Honfleur (base de données GIP Seine aval)

Les données de calcium restent comprises entre 85 et 140 mg/l dans un gradient de salinité important d'où l'utilisation des échelles logarithmiques. Cette gamme de variation n'est pas significativement différente de celle observée dans les eaux de l'aquifère de la craie. Le calcium ne pourra donc pas être utilisé comme marqueur géochimique des deux apports principaux dans les aquifères du remplissage estuarien.
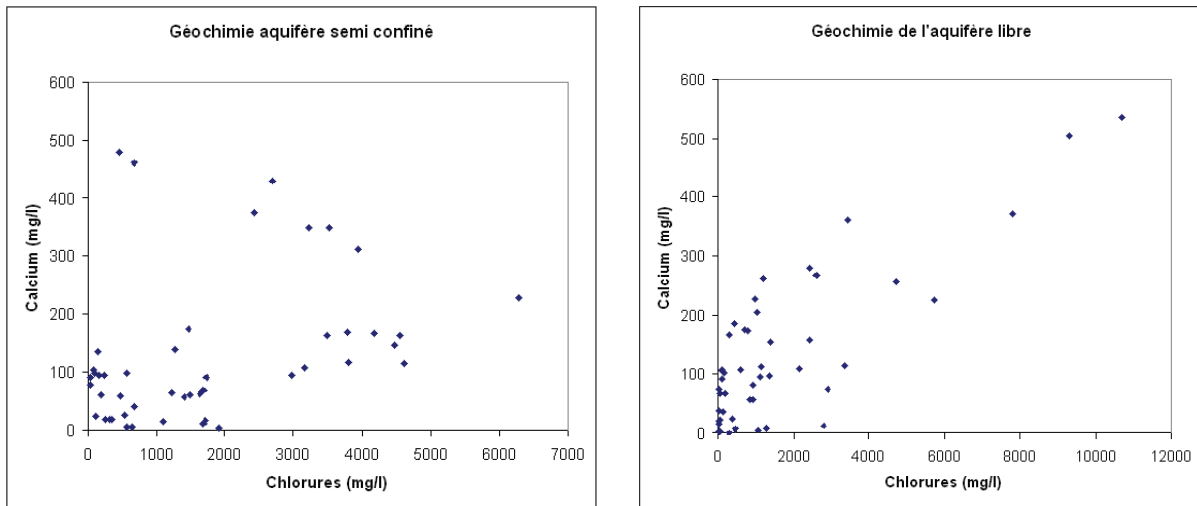
Le comportement du calcium dans ces aquifères poreux est toutefois analysé dans la Figure 39.

Figure 39. Comportement du Calcium dans les aquifères poreux semi confiné et non confiné

Dans les deux cas, la gamme de variation des teneurs en Calcium des eaux prélevées dans les piézomètres est beaucoup plus importante que dans les apports des deux masses d'eau principales. Cette extrême variabilité des teneurs en calcium rapportées aux fluctuations des teneurs en Chlorures témoigne de l'instabilité des équilibres calco-carboniques dans les eaux de ces aquifères poreux : précipitation (baisse des teneurs) /solubilisation (augmentation des teneurs). L'hétérogénéité physique et physicochimique de ces milieux poreux peu à moyennement carbonatés (par rapport aux écoulements dans l'aquifère de la craie totalement carbonaté) est responsable de cette instabilité des équilibres chimiques. Il est à noter que les résultats obtenu dans l'aquifère des graves de fond sont plus fortement hétérogènes.
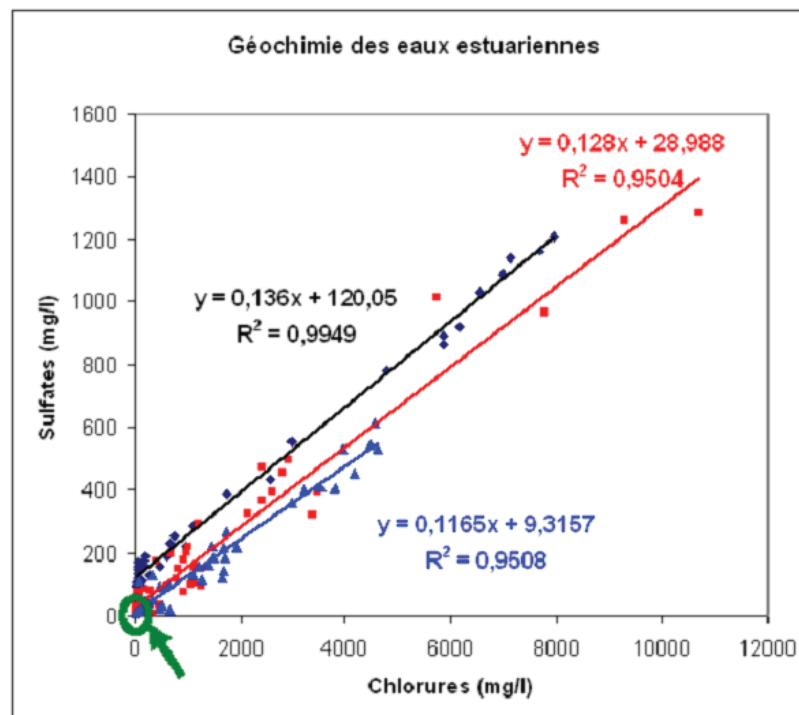
Ce constat rend délicate l'utilisation des modèles géochimiques classiques utilisés pour analyser les mélanges à partir des ions majeurs. Considérant que les bicarbonates sont généralement en excès tant dans les eaux de l'aquifère de la craie que dans les eaux du gradient de salinité de l'estuaire, nous considérerons que les précipitations de sulfates de calcium sont relativement

négligeables ce qui permet de tester la modélisation des mélanges à partir des teneurs en Sulfates dans les eaux étudiées.

En compilant l'ensemble des données acquises dans les diverses masses d'eau, la Figure 40 permet d'illustrer le comportement des sulfates en fonction des teneurs en chlorures.

Dans les eaux de l'aquifère côtier de la craie du secteur étudié, les teneurs en chlorures sont de l'ordre de 31 +/- 4 mg/l tandis que les teneurs en sulfates sont de l'ordre de 10 +/- 3 mg/l ce qui est englobé par le cercle vert de la Figure 40. Ces deux concentrations sont donc relativement stables et les points n'ont pas été reportés puisque se chevauchant, à l'échelle de ce graphique.

Pour ce qui est du gradient estuarien dans le chenal Seine (losanges bleu foncés et relation figurée en trait noir épais), nous représentons ici le scénario moyen le plus fréquemment observé qui fournit la relation entre l'augmentation des sulfates et celle des chlorures, élément conservatif utilisé aussi pour illustrer l'augmentation de conductivité électrique (paramètre qui peut être abordé par les mesures géophysiques in situ qui présente aussi une excellente corrélation avec les chlorures) ou de salinité.

Géochimie des eaux estuariennes

$y = 0,128x + 28,988$
$R^2 = 0,9504$

$y = 0,136x + 120,05$
$R^2 = 0,9949$

$y = 0,1165x + 9,3157$
$R^2 = 0,9508$

Figure 40.- Relations Sulfates/Chlorures dans l'ensemble des masses d'eau de la zone d'étude.

Sachant que les eaux de ce gradient correspond à un mélange déjà complexe impliquant les apports marins, l'influence des fluctuations du débit de la Seine et celle, également complexe,

des apports souterrains latéraux à la Seine en aval de Rouen notamment, nous proposons l'équation suivante qui a pour but d'optimiser les conditions moyennes de mélange :

$$[Sulfates] = 0,14\ [Chlorures] + 120$$

En ce qui concerne les eaux de l'aquifère semi confiné des graves de fond, la projection des données qualitométriques de l'ensemble des piézomètres longs (triangles bleus) fournit la relation sulfates/chlorures suivante (trait épais bleu et équation en bleu) :

$$[Sulfates] = 0,12\ [Chlorures] + 9$$

L'origine de ce nuage de point prend bien naissance dans la gamme des teneurs des eaux de l'aquifère de la craie du secteur étudié et la pente de la relation n'est pas très différente de la précédente mais témoigne néanmoins d'une convergence vers la tendance observée dans le gradient de salinité du chenal.

En ce qui concerne les eaux de l'aquifère non confiné, la projection des données qualitométriques de l'ensemble des piézomètres moyens (rectangles rouges) fournit la relation sulfates/chlorures suivante (trait épais rouge et équation en rouge) :

$$[Sulfates] = 0,13\ [Chlorures] + 29$$

L'origine de ce nuage de point s'inscrit également dans la gamme des teneurs des eaux de l'aquifère de la craie du secteur étudié et la pente de la relation n'est pas significativement différente de celle obtenue dans le gradient de mélange estuarien de surface.

Dans ces équations de mélange du type y = ax + b, le paramètre b représente la teneur en sulfates pour des teneurs en chlorures tendant vers 0. La valeur de l'ordre de 9 +/- 3 mg/l obtenue pour l'aquifère semi confiné converge bien vers les valeurs observées dans l'aquifère de la craie. L'aquifère des graves de fond est donc presque exclusivement soumis à des échanges latéraux entre l'aquifère de la craie et les eaux du chenal.

La valeur de l'ordre de 29 +/- 4 mg/l obtenue pour l'aquifère non confiné reste inférieure à celle observée dans le gradient estuarien de surface (de l'ordre de 120 +/- 5 mg/l) ; elle est intermédiaire entre les deux courbes de mélange et témoigne globalement de l'influence des

vannages qui permettent une inondation partielle des zones humides par des eaux estuariennes. Cette influence d'apports de surface d'eaux estuariennes se retrouve préférentiellement dans les zones concernées par ces vannages ainsi qu'à proximité du grand canal. Localement, ce comportement permettra d'évaluer l'influence de ces apports de surface par rapport aux données observées. C'est une approche qui permettra d'aborder l'hétérogénéité spatiale des différents apports dans l'aquifère libre.

A cet ensemble d'interactions, il faut ajouter les apports pluviaux qui s'infiltrent vers cet aquifère non confiné. Les teneurs moyennes en chlorures sont inférieures à 5 mg/l avec des pics exceptionnels qui ne dépassent guère 15 mg/l qui sont inférieures à celle de l'aquifère de la craie. Les teneurs en sulfates sont de l'ordre de 1 mg/l et celles de calcium sont en moyenne inférieures à 1 mg/l. Lorsqu'on prend en compte les données géochimiques de l'aquifère non confiné du remplissage sableux, nous devons convenir que l'effet de dilution par les eaux pluviales est peu significatif et que ce stock est en grande partie compensé par la consommation en eau de la végétation de ces zones humides. Conformément aux bilans réalisés par les écologues des années 70 (notamment dans l'ouvrage de F. Ramade), la consommation en eau de la végétation des zones humides peut être supérieure à 1000 mm/an ce qui est, localement, supérieure aux bilans des pluies annuelles efficaces.

Conclusion

Les analyses géochimiques ont également apportées des informations sur des paires de piézomètres ou des piézomètres isolés se distinguant des autres. De manière générale et quelque soit la période de l'année durant laquelle sont menées les campagnes, plusieurs piézomètres présentent des résultats en contradiction avec la majorité.

Lors de la campagne de janvier, les paires de piézomètres 3798/3799, 3800/3801, 3804/3805 3814/3815 ainsi que les piézomètres 3788, 3811, 3806 ce sont distingués des autres piézomètres quelque soit la méthode d'analyse utilisée.

Lors de la campagne de juin, les paires (ou trio) de piézomètres 3798/3799, 3800/3801/3521, 3804/3805, 3814/3815 ainsi que les piézomètres 3788, 3796, 3809 et 3811 se sont distingués des autres piézomètres quelque soit la méthode d'analyse utilisée.

Si l'on croise en plus les résultats du tritium pour lesquels ce sont les paires de piézomètres 3804/3805, 3796/3797, 3792/3793, 3788/3789, 3784/3785 qui se distinguent, il paraît évident que le marais du Hode a un fonctionnement hydraulique d'une grande hétérogénéité.

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ANNEXE I

Geostatistical inverse modeling of the transmissivity field of a heterogeneous alluvial aquifer under tidal influence

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Running title: Transmissivity field of an alluvial aquifer.

Key words: hydraulic tomography, tide response, alluvial aquifer, inversion, transmissivity, inverse problem, pilot points.

Abstract. The hydraulic head of coastal aquifers oscillates with tides because of their poroelastic compressibility. The record of these fluctuations can be used to reconstruct the transmissivity field. The pilot points method is used here, in a stochastic framework using the Markov chain Monte Carlo (McMC) sampler, to determine the hydraulic transmissivity field of an alluvial aquifer connected to the estuary of a river. This approach consists in inverting the hydraulic head response to fluctuations of the tide. First, our algorithm is successfully benchmarked on a synthetic case study. Then, we present a real case study in which both a semi-confined aquifer and a shallow unconfined aquifer are connected to the estuary of the Seine River in the western-north of France. These aquifers experience strong tidal variations up to 8.4 meters. This test site is equipped with a series of seventeen wells in which the temporal fluctuations of water heads associated with megatidal forcing were recorded during several years. In addition to the use of the pilot points method, a zoning parameterization with forty five zones is used to determine the vertical leakage coefficient from the semi-confined aquifer into the shallow unconfined aquifer. The distribution of the vertical leakage coefficient is determined by inverted the hydraulic head at a set of wells localized in the semi-confined aquifer. We determine jointly the distribution of the hydraulic transmissivity and the vertical leakage from the semi-confined into the unconfined aquifer using 72 pilot points. The storage coefficient is assumed to be constant over the whole domain. Its value is determined from two pumping tests that show a consistent value for this parameter ($\log_{10}S$=-5 and 0.11 as a variance). The spatial distribution of the hydraulic transmissivity agree qualitatively with the soil texture variations observed in the semi-confined aquifer and derived from the analysis of core samples.

1. Introduction

Reliable predictions of ground water resources require accurate information related to the distribution of hydraulic conductivity or transmissivity (hydraulic conductivity times the thickness of the aquifer) and the storage coefficient. Historically, the transmissivity and storage coefficient were based on pumping test analysis (Theis, 1935). More recently, hydraulic tomography was developed to integrate more information regarding the head fluctuations from both a set of pumping / injection wells plus observations wells, in order to characterize the distribution of these properties in heterogenous aquifers (e.g., Yeh and Liu, 2000; Zhu and Yeh, 2005). Usually in alluvial aquifers, the transmissivity can vary over 5 to 10 orders of magnitude while the storage coefficient varies over one or two decades. It is therefore customary to keep the storage coefficient constant and to invert the transmissivity field (Younger, 1993)

Hydrogeologists have developed various approaches to perform the inverse problem fitting hydraulic head data to invert the distribution of the transmissivity field. In the so-called the pilot points method, the number of the parameter to estimate is decreased by identifying a few key-locations in the aquifer where the values of the transmissivity can be inverted to reproduce the observed head measurements (de Marsily et al. 1984). This is done to the expense of assuming certain spatial statistical characteristic of the transmissivity field. In this approach, the values and positions of the pilot points are used to perturb the transmissivity field from the prior pattern, which is determined for instance from the traditional inversion of a set of single pumping tests. The optimal values of the pilot points and their positions used to reproduce the observations of the hydraulic heads are traditionally determined via a deterministic optimization algorithm. Therefore, both the values and locations of the pilot points are considered as the parameters of calibration. This process, combining geostatistical characteristics of the aquifers (like the correlation lengths) and optimization, has been successfully applied to various hydrogeological problems (LaVenue and Pickens, 1992; RamaRao et al., 1995). In order to

reduce the instability resulting from the choice of the location of the pilot points and over-parameterization of the problem, a number of approaches have been proposed during the two last decades to choose the position of the pilot points (see Certes and de Marsily, 1991; LaVenue et al., 1995; Cooley, 2000; Medina and Carrera, 2003; Doherty, 2003; Carrera et al,. 2005). In order to take into account the non-uniqueness of the solution of this inverse problem, Sahuquillo et al. (1992) and Rama Rao et al. (1995) used conditional simulations in the optimization process in order to obtain an ensemble of solutions with the same likelihood.

In the present work, we are interested to determine the transmissivity field of an alluvial heterogeneous semi-confined aquifer connected to the estuary of a river under tidal influence. For coastal aquifers, pumping tests are not always desirable because they may accelerate seawater intrusion (Chapuis et al., 2006). As the hydraulic heads of coastal aquifers fluctuates naturally because of tidal forcing, it is tempting to use these natural head fluctuations as a source of excitation to estimate hydraulic parameters (Jacob, 1950; Ferris, 1951; Jiao and Tang, 1999; Li, et al.,2002). Battle-Aguiar (2008) combined pumping/injection tests, tracer experiments, and monitoring of the groundwater /river interaction to characterize the transport properties of an alluvial aquifer. In the so-called "tidal response approach", researchers have analyzed the head fluctuations in a set of wells in response to changes in the sea water level caused by the tides (Jacob, 1950; Gregg, 1966; Carr and van der Kamp 1969; Nielsen, 1990 and Li, et al., 2007). This approach requires therefore to have times series of sea levels and the corresponding time series of hydraulic heads recorded at one or several wells affected by tidal variations. Using this information, inverse modeling is used to estimate the mean hydraulic properties (i.e., a mean hydraulic conductivity or transmissivity and a mean storativity) of coastal aquifers (Jiao and Tang, 1999 and Jeng. et al., 2002). However, this interpretation process is based on a simple analytical 1D equation that describes the groundwater response of a homogenous and isotropic confined aquifer to tidal oscillations imposed as a boundary condition (Todd, 1959). These assumptions are not appropriate for heterogenous aquifers and a more general approach is

required. Recently, Alcolea et al. (2007, 2009) presented an attempt to overcome the previous problem to characterize the heterogeneity of the transmissivity field of a coastal aquifer by using the pilot points method discussed above.

In the present paper, we are interested to expand this approach to the case of two aquifers connected to the estuary of a river under tidal influence. One of these aquifers is a semi-confined aquifer, which is locally connected to a shallow unconfined aquifer. So in addition to the transmissivity field, we have also to invert the vertical leakage coefficient from the semi-confined aquifer into the shallow unconfined aquifer using the hydraulic head responses to tide variations observed at a set of wells located in both aquifers. We propose an approach integrating also the impact of the regional flow on the observed head fluctuations used within the inversion process. Another improvement on previous modeling efforts is that while previous authors used deterministic methods to choose the values of the transmissivity at the pilot points, we place the pilot points on a regular grid and we use a stochastic approach for taking into account the non-uniqueness of the solution of the inverse problem. This stochastic approach is based on a variant of the Markov chain Monte Carlo (McMC) sampling approach called the Adaptative Metroplis Algorithm (AMA) Therefore there are three novel aspects in our work: (1) We are dealing with two alluvial aquifers with a leakage from the semi-confined aquifer to the unconfined aquifer, (2) we are accounting for the regional ground water flow on the head response, and (3) we are using a stochastic method rather than a deterministic method in performing the inversion, which is a computer intensive approach but it has the advantage to account for the non-uniqueness of the inverse problem.

2. Forward Problem

We consider two alluvial aquifers in interaction with the estuary of a river under tidal influence as sketched in Figure 1. These aquifers are (1) a deeper semi-confined aquifer and (2) a shallow unconfined aquifer. These aquifers are separated by an aquitard that is however

discontinuous. We consider therefore some localized seepage flow from the deeper semi-confined aquifer to the unconfined aquifer. This localized flow will be modeled below by a vertical leakage contribution $L(h_s - h)$ where L (in s^{-1}) denotes the vertical specific leakage coefficient, h (in m) represents the hydraulic head of the semi-confined aquifer and h_s (in m) represents the piezometric level of the unconfined aquifer. Note that in the following we will consider that the tidal head fluctuations in the unconfined aquifer are negligible compared to head fluctuations in the semi-confined aquifer (Figure 2). This assumption has been proven to be valid by several authors (e.g., White and Roberts, 1994) except in the immediate vicinity of the estuary. In the vicinity of the estuary, the unconfined aquifer oscillates also with tidal variations via the vertical seepage from the semi-confined aquifer. In the case study presented later below, this is a very good assumption because of (i) the low hydraulic transmissivity of the unconfined aquifer and (ii) the presence of a dam settled in the unconfined zone that absorbs the propagation of the tide wave in the shallow aquifer (see Figure 1)

The semi-confined aquifer can be subdivided themselves into two sub-domains Ω_1 and Ω_2. The sub-domain Ω_1 corresponds to the immersed parts of this aquifer while Ω_2 corresponds to the remaining (inland) part. We are interested to model the influence of the tidal force upon the hydraulic head in the sub-domain Ω_2 of the aquifer. Traditionally, in this situation, the tidal force is imposed as a boundary condition at the interface between the river and the aquifer. This boundary condition is then used to solve the diffusion equation for the hydraulic head. The numerical solution of this flow field is considered below as our forward problem. This forward model will be iterated later in the inverse modeling to determine the spatial distribution of the transmissivity field of the aquifer.

The hydraulic head h (in m) is therefore main variable to solve for the forward model. The hydraulic diffusivity depends on the ratio between the specific storage and the hydraulic transmissivity. The relationship between the pore fluid pressure p (in Pa) and the head h is

$p = \rho_f g \ h$ - z where ρ_f (in kg m^{-3}) is the mass density of the pore water (considered constant),

g (in m s^{-2}) is the gravity acceleration, and z (in m) is the constant elevation from which the hydraulic heads are counted out. For the semi-confined domain Ω_2 of Figure 1, the flow equation is therefore,

$$S_\alpha \frac{\partial h}{\partial t} - \nabla \cdot (T \nabla h) = L(h_s - h) , \tag{1}$$

where T and S_α denote the hydraulic transmissivity (in m^2 s^{-1}) and the specific storage of the semi-confined aquifer, respectively. The diffusion equation, Eq. (1), is solved with the following boundary and initial conditions:

$$h(x, y, 0) = h_0(x, y) , \text{ at } t = 0, \tag{2}$$
$$h(x, y, t) = h_{mean} + h_A \sin(\omega t) , \text{ at } \Gamma_0 , \text{ and } x = 0, \tag{3}$$
$$h(x, y) = h_1 \text{ at } \Gamma_1 , \tag{4}$$
$$T \nabla h(x, y, t) \cdot \mathbf{n} = 0 \, m\overset{2}{}s^{-1} , \text{ at } \Gamma_2 \text{ and } \Gamma_3 , \tag{5}$$

where Γ_0, Γ_1, Γ_2 and Γ_3 are fourth boundaries defined below, h_{mean} corresponds to the mean elevation of the river, the maximum amplitude of the fluctuation is denoted as $h_A, \omega = 2\pi/T_0$ denoted the angular frequency for a period T_0, and $h_0(x, y)$ represents the initial hydraulic head associated with the regional flow (see below). In this paper, the vertical leakage coefficient L from the semi-confined aquifer into the unconfined aquifer is zero except in some limited area where the leakage was observed. In our approach, we will estimate both the transmissivity field and the distribution of the coefficient L, a problem that has never been attempted as far as we know.

In Eq. (3), the boundary Γ_0 denotes the interface between the river and the semi-confined aquifer. In our model Γ_0 is assumed to be a vertical plane (see the left-hand side of Figure 1). Therefore, the boundary condition for the head at this boundary is expressed as a harmonic oscillation of the hydraulic head, see Eq. (3). The hydraulic head imposed at the right-hand side boundary of the domain, Γ_1, is assumed to be far away from the area of the tidal

influence. The boundary condition at Γ_1, Eq. (4), is due to the regional flow contribution shown in Figure 1. In the real case study discussed below, this regional ground water flow is coming from a karstified chalk aquifer in contact with the aquifers (Figure 1). All the other boundaries (Γ_2 and Γ_3), see Eq. (5), are assumed to be impervious (no-flow boundaries).

We recast our problem with the new hydraulic head $H = h - h_0$ which is due only to tidal component of the flow. Using this change of variables in Eqs. (1) to (5), we can rewrite our boundary-value problem as:

$$S_\alpha \frac{\partial H}{\partial t} - \nabla \cdot (T\nabla H) = L(h_s - H - h_0) + \nabla \cdot (T\nabla h_0) , \tag{6}$$

$$H(x, y, 0) = 0 , \text{ at } t = 0, \tag{7}$$

$$H(x, y, t) = h_A \sin(\omega t) , \text{ at } \Gamma_0 , \text{ and } x = 0, \tag{8}$$

$$H(x, y) = 0 \text{ at } \Gamma_1 , \tag{9}$$

$$T\nabla H(x, y, t) \cdot \mathbf{n} = 0 \, m \, s^{-1} \, \Gamma_2 , \text{ and } \Gamma_3 . \tag{10}$$

In this new formulation of the forward problem, the regional flow appears as a hydraulic sink in the flow equation (see the last term of Eq. 6). For heterogeneous aquifers, the solution of the hydraulic diffusion equation can be performed with numerical methods such as the finite element method used below. Only under the restrictive assumptions of a single, homogenous, horizontal, and semi-infinite aquifer; the hydraulic diffusion equation has an analytical solution given by (Ferris, 1951):

$$H(x) = h_A \exp\left(-x\sqrt{\pi S/T_0 T}\right) \sin\left(\omega t - x\sqrt{\pi S/T_0 T}\right) , \tag{11}$$

where x is the horizontal distance from the shoreline. This analytical solution has been broadly used in the past to estimate the transmissivity and storage coefficients in the literature. Eq. (11) implies that the amplitude of the hydraulic head fluctuations decreases as an exponential function of x and depends on the hydraulic diffusivity $D = T/S$. The phase lag depends on both the position

x and on the hydraulic diffusivity. Considering a monitoring well with a negligible phase or time lag, Carr and Van der Kamp (1969) defined the tidal efficiency γ as:

$$\gamma = \left(\frac{\partial H}{\partial h_t} \right) = \exp\left(-x\sqrt{\pi S/T_0 T} \right),$$

(12)

where h_t represents the tidal oscillations recorded by tidal gauges. We can use these estimates of the hydraulic diffusivity to build a prior kriged field for this parameter or alternatively for the transmissivity field assuming that the specific storage is independently determined. This field can be used, in turn, to initiate the inversion algorithm using the pilot point approach.

3. Geostatistical Inverse Modeling

The pilot points technique, introduced initially by De Marsily et al. (1984), represents one of the most popular and reliable methods for solving the ground water flow inverse problem. In the last decades, this concept has been broadly used and the version proposed by De Marsily et al. (1984) has been modified. The pilot points method consists in generating an initial kriged transmissivity field from a set of local measured values of transmissivity, determined from local pumping tests for instance. Then, this field is perturbed to reproduce the head measurements with a set of complementary transmissivity data at a set of selected positions in the aquifer referred as the pilot points.

The choice of the number and location of the pilot points is a key-step to stabilize the inverse modeling. De Marsily et al. (1984) selected empirically the positions of the pilot points. In an alternative approach developed by Sahuquillo et al. (1992) and Gomez-Hernandez et al. (1997), the pilot points are laid out at the same time on a pseudo-regular grid and they suggest to modify the values of all pilot points at the same time. This technique is known as the "master points" approach.

We propose in the next section a new approach, which follows the procedure developed by Alcolea et al. (2006) with a difference in the optimization step. We use the Markov chain

Monte Carlo (McMC) approach to design the optimal values of the transmissivities instead of using a Gauss-Newton (or gradient) approach (Fu and Gomez-Hernandez, 2009 and Oliver et al, 1997). The McMC sampler is used to find the hydraulic parameters able to reproduce the piezometric observations. This approach is therefore more expensive in computer power but allow to handle the non-unicity of the inverse problem by probing the most likely model.

The procedure used below consists in five-steps:

Step 1: We estimate the geostatistical model parameters such as the variogram function, $\gamma\ h_{xy}, p$ where h_{xy} is a separation vector and p a vector of variogram parameters (such as the sill and the anisotropy angle) and the variance. These parameters are determined through the analysis of available measurements and observations (possibly geological datasets, hydrogeological information, and/or geophysical information). In some cases, the number of observations can be limited and there are therefore some significant uncertainties in establishing a reliable prior variogram. The choice of the geostatistical model parameters used to represent the variogram will be sampled during the McMC process inside the intervals of uncertainties of the experimental variogram.

Step 2: We construct the transmissivity field from the superposition of the two kriged fields:

$$f(\mathbf{x}) = f_{\mathrm{I}}(\mathbf{x}) + f_{\mathrm{P}}(\mathbf{x}), \tag{13}$$

where the first field $f_{\mathrm{I}}(\mathbf{x})$ is obtained through the kriging of the available transmissivity measurements using the experimental variogram. These transmissivity data will be determined from local hydraulic pumping tests. We note that this initial transmissivity field does not necessarily reproduce the hydraulic head data. This first field $f_{\mathrm{I}}(\mathbf{x})$ can be expressed as a linear interpolation from N_o transmissivity estimates:

$$f_{\mathrm{I}}(\mathbf{x}) = \sum_{i=1}^{No} \lambda_i^k(\mathbf{x}) f_{\mathrm{i}}^*, \tag{14}$$

where **x** is the locations where the transmissivities are estimated and λ_i^k are the kriging weights

between the interpolation point k and f_i^* the measurement at point i. The second field in Eq. (13),

$f_P(\mathbf{x})$, is used to ensure the perturbation of the initial field to honor the hydraulic head data

associated with the observed tidal response in the semi-confined aquifer. This perturbation field

is also obtained by a kriging estimate from a set of pilot points values of the transmissivity. The

variogram parameters used to build this second field are unknowns and iteratively estimated in

the inverse modeling. In our case, the pilot points are localized on a regular grid following the

idea expressed by Gomez-Hernandez et al. (1997). The unknown values of the transmissivity at

the pilot points should be assessed with an iterative optimization process or a stochastic

approach. The perturbation $f_P(\mathbf{x})$ can be expressed as a linear combination of the N_p model

parameters with the kriging weights λ_i^{pp} at the point k and p_i^* the value of the tranmissivity at the

pilot points:

$$f_P(\mathbf{x}) = \sum_{i=1}^{Np} \lambda_i^{pp}(\mathbf{x}) p_i^* . \tag{15}$$

The main idea of this approach is to find, with a stochastic approach, an optimal perturbation

field able to reproduce the hydraulic head measurements.

Step 3: We solve the forward problem of ground water flow, which allows connecting the

spatial distribution of the transmissivity field $f(\mathbf{x})$ and hydraulic head variations H. The

forward problem consists in solving the flow (diffusion) equation with the finite element method

for a given transmissivity field resulting from the Step 2 above and with suitable boundary and

initial conditions (assumed to be perfectly known). Using the change of variables outlined above

in Section 2, we consider that the flow of the aquifer is driven by the tidal variations imposed by

the harmonic boundary conditions.

Step 4: In a probabilistic framework, the inverse problem maximizes the conditional

probability density of the model **m** *(Mx1)* given the observed data vector **d***(Nx1)*. This

corresponds to find the best model **m** respecting the constraints posed by the prior model and minimizing the difference between the observed and predicted data. We note $P_0(\mathbf{m})$ is the prior probability density of parameters **m** and $P(\mathbf{d}|\mathbf{m})$ denotes the probability density corresponding to the data for **m** fixed. The posteriori probability density $\pi(\mathbf{m}|\mathbf{d})$ of the model parameters **m** given the data **d** is obtained using Bayes formula:

$$\pi(\mathbf{m}|\mathbf{d}) \propto P(\mathbf{d}|\mathbf{m})P_0(\mathbf{m}) , \tag{16}$$

$$P(\mathbf{d}/\mathbf{m}) = \frac{1}{[(2\pi)^N \det C_d]^{1/2}} \exp\left[-\frac{1}{2}(h(\mathbf{m})-\mathbf{d})^T C_d^{-1}(h(\mathbf{m})-\mathbf{d}) \right], \tag{17}$$

$$P_0(\mathbf{m}) = \frac{1}{[(2\pi)^M det C_m]^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{m}-\mathbf{m}_{prior})^T C_m^{-1}(\mathbf{m}-\mathbf{m}_{prior}) \right], \tag{18}$$

$$\mathbf{m} = \left[\mathbf{m_p}, \mathbf{m_L}, \mathbf{m_v} \right], \tag{19}$$

where **m** denotes the model of unknown model parameters to be explored during the inverse process. This model vector is actually a composite of the vector of the perturbation fields $\mathbf{m_p}$ and the vector $\mathbf{m_L}$ denoting the distribution of the vertical leakage coefficient. It also includes the set of variogram parameters $\mathbf{m_v}$ involved in the kriging routine of the pilot points.

The vector $h(\mathbf{m})$ denotes the numerical estimates of the hydraulic heads using the value of the transmissivities, vertical leakage coefficients, and variogram parameters. The matrix C_{d_i} ($N{\times}N$) denotes a diagonal covariance matrix to account for the noise-to-signal ratio of the data in the inversion. The vector $\mathbf{m}_{prior}$ *(Mx1)* denotes the prior value of all the model parameters and the matrix C_m ($M{\times}M$) denotes the model (diagonal) covariance matrix incorporating the uncertainties related of the prior model of material properties. The prior model is used to overcome unrealistic sampling of the distribution of the model parameters.

To quantify the distribution of the leakage coefficient, we cannot use the pilot point parameterization. Indeed, this parameter may be discontinuous and therefore there is no way to

estimate a variogram based on initial data for this parameter. We use therefore a zoning parametrization approach as the tool applied to estimate the distribution of the vertical leakage instead of the pilot point approach. The choice of the zoning parametrization will be explained in more details below in Section 4.3.

Step 5: Rather than using a deterministic approach to reconstruct the flow properties, we use a McMC approach to sample, from their posterior probably density, the unknowns model parameters. This is done by generating: (i) the values of the transmissivity at the pilot points to build the perturbation field $f_P(\mathbf{x})$, which is added to the initial transmissivity field (this sum of the field is conditioned to reproduce the observed head data fluctuations), (ii) the vertical leakage coefficient between the two aquifers, and (iii) the variogram parameters for the transmissivity field. The relationship connecting the data vector $\mathbf{d}$ and the model vector $\mathbf{m}$ is non-linear. In this case, using a deterministic method is complex and sometimes unreliable due to the difficulty of getting a linearized estimate of the objective function which is based on approximations of the derivative using a Taylor expansion. The use of a McMC sampler allows to determine the values of the hydraulic parameters (T and L) without computing derivatives of the objective function.

The McMC algorithms consist of random walks where different values of the transmissivity at the pilot points, the values of the leakage coefficient, and the values of the variogram coefficients are sampled and where the choice of the next state depends only on the value of the current state (Sen and Stoffa 1995; Mosegaard and Tarantola 1995; Gelman et al. 1996; Malinverno 2002; Jardani et al., 2010). After an initial period, the random walker moves toward the highest probability regions for the model parameters. Once the chain has converged, the sampler returns a number of model vectors sampling the posterior probability density $\pi(\mathbf{m}|\mathbf{d})$.

In the present paper, we implemented a variant of the McMC algorithm called the Adaptive Metropolis Algorithm (AMA, see Haario et al. 2001, 2004). We assume that ($\mathbf{m}^0$, …,

$\mathbf{m}^{i-1}$) denotes a sampled ensemble of the transmissivity values at the pilot points, vertical leakage coefficients, and variogram parameters ($\mathbf{m}^0$ denotes the initial state used to start the Markov chain). The next candidate **m'** is sampled from a Gaussian proposal distribution *q*, which is updated at each iteration from the past collection of realizations. This approach uses the mean at the current state and a covariance matrix $\mathbf{C}^i$ that depends on the history of the Markov chain according to:

$$C^i = s_n K^i + s_n \varepsilon I_n, \tag{20}$$

where $K^i = Cov(\mathbf{m}^0, ..., \mathbf{m}^{i-1})$ denotes the covariance matrix, I_n is the $n \times n$ identity matrix, ε is a small positive number to avoid the singularity of the covariance matrix, and $s_n = (2.4)^2 / n$ is a scaling factor that depends on the number of the unknowns parameters *n*. The candidate state **m'** is selected with the following probability:

$$\alpha\ \mathbf{m}^{i-1}, \mathbf{m'} = \min\left(1, \frac{\pi\ \mathbf{m'}}{\pi\ \mathbf{m}^{i-1}}\right). \tag{21}$$

If the candidate realization is accepted, we consider $\mathbf{m}^i = \mathbf{m'}$ otherwise $\mathbf{m}^i = \mathbf{m}^{i-1}$. Memory mechanism of the McMC algorithms (that makes the chain staying in the high posterior probability regions of the model space) are responsible for a greater efficiency of the algorithm in comparison with the more classical Monte Carlo sampling methods for which the models are independently chosen and tested against the observations (Sternberg, 1979).

4. Case study

4.1. Description of the Test Site

The test site is located in the Seine River estuary in the northwest of France. (Figure 3a). This test site is equipped with a couple of seventeen piezometers settled at two different depths in order to record the temporal fluctuations of the hydraulic heads in both the semi-confined and

the unconfined aquifers (Figure 1). The site is affected by strong tidal forcing with an amplitude reaching 8.4 meters, which is quite a strong head changes imposed to the aquifers.

The alluvial fill was formed on an anticline with a clayey substratum at the foot of the cliff and sandy clay at the remaining alluvial plain (Figure 1). In the lower-Seine valley, the fluvial deposits formations are composed by three sedimentary sequences (see Figure 4 and Lesourd et al., 2001). The unconfined shallow aquifer extends from a depth of 8 m to a depth of 33 m. It consists mainly of poorly-sorted sediments including fine sands and localized silty clay. The material of this aquifer coarsens with depth being and medium to coarse sands are observed at the base of the aquifer. The semi-confined aquifer corresponds to the first alluvial deposit covering the substratum. Its thickness varies from 4 m to 16 m. It is formed by poorly to moderately sorted sands and gravels with a considerable increase in the grain size with depth. The grain sizes, the percentage of the sands and gravels in the sediment and their sorting are the possible factors controlling both the values of the porosity and the permeability of the semi-confined aquifer. The semi-confined and unconfined aquifers are separated by a thin and discontinuous silty-peat formation playing therefore the role of a discontinuous aquitard. The sediment of this aquitard has very small grain sizes and therefore very low hydraulic conductivities (on the order of 10^{-8} m s^{-1} as checked through unpublished laboratory measurements). The absence of this aquitard in some areas involves some hydraulic seepage from the lower to the upper aquifers. According to boreholes logs, the thickness of this aquitard varies from 0 to 12 meters.

The hydrogeological studies showed that both aquifers are fed mainly by the karst aquifer existing in the calcareous rock through the lateral leakage of this karst aquifer especially during flood events. The analysis of the head data chronicles revealed that the alluvial plain is connected with the Seine River. Consequently, we identified a tidal periodicity on the piezometric fluctuations. We note that the site is however protected against flooding (because of the flood events of the Seine River) by a dam whose base is built on the surface deposits of the

bank river. The position of this dam is shown in Figure 8 and its presence is therefore important in the hydraulic characterization of the aquifers. The dam is able to promote the vertical drainage from the semi-confined to unconfined aquifer with a drop of hydraulic pressure of the surface aquifer compared to that of the semi confined aquifer.

4.2. Validation with a Synthetic Case Scenario

We present in this section a synthetic example to illustrate the efficiency of the inverse approach outlined above in Section 3. As previously discussed, the hydraulic head response to the tidal force fluctuations depends mainly on the distance to the river. This makes the sensitivity of the characterization approach dependent on the river geometry. Consequently, the geometry of the synthetic case proposed here is similar to the geometry of the case study discussed in more details in Section 4.3 (see Figure 5).

The first boundary of interest in modeling the semi-confined aquifer, Ω_a, is located between the chalk cliff and the Seine River itself. Actually this boundary is located quite far from the area of interest in order to avoid to take into account the tidal fluctuations at this boundary. The second boundary, denoted as Ω_k below, is located between the north boundary and the karstic chalk cliff. We use a constant transmissivity of 10^{-3} m^2 s^{-1} to simulate this area (this value is inferred from pumping tests, Hanin 2010).

In this synthetic example, the ''true'' Log transmissivity field used for the synthetic test is chosen to be heterogeneous and was generated by using the sequential Gaussian simulation (SGSIM) provided by Deutsch and Journel (1998) (Figure 5a). We use a spherical anisotropic variogram (with sill of $1(Log_{10}[m^2s^{-1}])^2$, zero nugget, horizontal and vertical ranges equal to 4000 m and 3000 m, respectively). The values of the ''true'' resulting $\log_{10}$ T-field range from -6 to -2, $[\log_{10}, m^2 s^{-1})]$ (see Figure 5a). The spatial distribution of the transmissivity field is characterized by a lower transmissivity channel crossing the aquifer model. In this work, only the heterogeneity of the transmissivity field is explored. The storage coefficient S is assumed to

be constant and known over the whole domain. We use a value of 10^{-5} for S to satisfy to the hydrogeological conditions of the real case study. We also created a flow component due to a regional ground water flow $h_0(x, y)$ generated from the karstic aquifer. This head component is added to the tide oscillations (see Eq. 6 above). The regional head $h_0(x, y)$ is obtained by solving the groundwater flow equation in steady-state conditions under the following boundary conditions (see Figure 3): $h_k = 2.5$ m at the north boundary Γ_1 (the hydraulic head of karstic aquifer); $h_{mean} = 0$ m denotes the mean hydraulic head at the river /sediment boundary Γ_0 and the impervious boundary condition is applied at all the two other boundaries, Γ_2 and Γ_3.

To simulate the harmonic fluctuations of the level of the river, we use an amplitude of $h_0 = 4$ m and a tidal period of $T_0 = 12$ hours and 24 minutes. This boundary condition is imposed at the western boundary Γ_0 of the aquifer. A set of the piezometeric head data are sampled at seventeen well locations (see their location in Figure 3b). This information corresponds to the basic data used to map the transmissivity field in the inverse process. The amplitude analysis of the hydraulic head data are reported in Figure 7. We can clearly distinguish between the head data sampled in the high transmissivity area from those sampled in the low-permeability areas. For instance, in the high transmissivity area, the amplitude of the hydraulic head data recorded in Well #3 reaches an amplitude of 3.25 m. This value is by itself an indication of the high transmissivity of the area around Well#3. This is indeed required for the tidal energy to propagate to this well.

On the other hand, in the low transmissivity zone, the hydraulic head variations induced by the tidal oscillations are characterized by low amplitudes in the hydraulic head fluctuations. For example, Well #13 exhibits a much smaller fluctuations ($\approx 1.1\,m$) by comparison with Well #3($\approx 6.5\,m$). Given that the distances of these two wells to the river are similar, this means that the transmissivity is smaller around Well #3. The analysis of the shape of the hydraulic head data

represents therefore the first useable indication about the transmissivity of the aquifer around the wells and can be therefore used to establish an initial log-transmissivity field to start the inverse process. Four measurements of $\log_{10}T$ were selected from the ''true'' field as observed transmissivity data (see Figure 5b). This sampling was located in such a way that the initial transmissivity field (calculated by ordinary kriging (see Figure 5b) was quite distinct from the ''true'' T-field (compare Figure. 5a to Figure 5b). We observe for instance that the lower $\log_{10}T$ area crossing the zone of interest is not accounted for in this initial pattern (see Figure 5b and compare it with the true field shown in Figure 5a). Thus, the reconstruction of true field from the hydraulic head data and initial model of transmissivity is heavily dependent on the calibration of the perturbation field by the perturbation of the transmissivity values at the pilot points. Regarding the number and choice of the locations of the pilot points, we use 72 pilot points to cover the area (materialized by the open circles in Figure 5c).

As explained above in Section 3 above, the AMA was adopted to reduce the difference between the observed and calculated harmonic hydraulic heads through the estimation of the transmissivities at the position of the pilot points. In this synthetic case study, we assume that the variogram parameters are known. In addition, to avoid unrealistic values of the transmissivity at the pilot points, we bound its sampling using the following proposal distributions for the ($\log_{10}T$): [-8, 0]. The parameters of the weighting matrix $\mathbf{C_d}$ are strongly dependent to the amplitude of the hydraulic heads. We used 27,0000 realizations to sample the posterior distribution with the AMA algorithm. The McMC converged after 22,000 realizations and therefore we used the last 5000 realizations to establish the posterior probability density of the model parameters. The value of the transmissivity at each pilot point is characterized by a posterior probability function determined by the distribution of the realization once the McMC algorithm has converged.

Figure 6 illustrates an example of the posterior probability function of the transmissitivity at a pilot point. The best values of the pilot points correspond to the highest probability of the

posterior probability distribution. These values have been replicated the hydraulic heads as proved by a comparison between the observed head data and the computed one (see Figure 7). The estimated transmissivity field is shown with a map of the associated standard deviation (given in Figure 5d). The map of the inverted transmissivity field provides the location of the permeable and less permeable regions of the aquifer in agreement with the "true" transmissivity field (compare Figure 5c with Figure 5a).

4.3. Application to a Case Study

The piezometric variations of the semi-confined aquifer were recorded on 17 wells. These head fluctuations include both the effect of the regional groundwater flow and the harmonic changes due to the tidal oscillations. In the present study, the piezometric data were recorded during 100 hours during which both the regional flow from the karstic aquifer and the vertical infiltration remained stable (no strong rain events and dramatic change in the mean level of the river). The effect of the regional flow within the alluvial zone Ω_a upon the hydraulic heads can be taken as an average value of the piezometric head data for some time windows. At the karstic boundary Ω_k, the hydraulic head imposed on the northern boundary is $h_k = 8$ m according to the karstic studies conducted on the site with a transmissivity of 10^{-3} m s^{-2} (from Hanin, 2010, see Figure 3). In order to determine only the hydraulic response associated with tidal oscillations and to avoid integrating the effects of horizontal regional flow, we corrected the hydraulic head data of the regional flow as explained in Section 2 above. This was done for each well by subtracting the mean value over time in order to detrend the data.

The effect of the residual response associated with the tidal effect is shown in Figure 10. As explained in Section 2, the amplitude of hydraulic head fluctuations in the semi-confined aquifer depends mainly on the shortest distance to the river and on the value of the hydraulic diffusivity of the aquifer between the shoreline and the well (Figure 8). In addition, as explained

in Section 2, the seepage between the semi-confined and unconfined aquifers influences the hydraulic head fluctuations in the semi-confined aquifer. The difference of the amplitudes of the heads located at the same distance from the river is therefore indicative of the heterogeneity of the transmissivity of the semi-confined aquifer (Figure 8). The amplitude analysis shows that there is (as expected) a general decrease of the amplitude with the distance from the Seine River. From these chronicles, we observed that the amplitude of the head variations is relatively small ($\leq 0.1\,m$) in the central part of the investigated area (at piezometers #3802 and #3804). At the opposite, in the western part of the site (at piezometers #3784, #3786, #3788, #3790, #3792 and #3794), the tidal wave is less attenuated and the head variations are relatively important($\geq 1\,m$) (see Figure 8).

Because of the dam, there is an area where the hydraulic head of the unconfined aquifer are lower compared to the semi-confined aquifer. Therefore because of the dam, the vertical infiltration becomes an important factor to consider in the analysis of the hydrogeological data. In this area, the head pressure chronicles recorded both in the unconfined aquifer and in the semi-confined aquifer indicates a significant presence of a vertical seepage from the semi-confined aquifer. For instance, Figure 11a shows that a low hydraulic head change ($\approx 5.75\,m$) is recorded in Wells #3811 in the unconfined aquifer while a higher hydraulic head change ($\approx 6.75\,m$) is recorded in the semi-confined aquifer in Wells #3810. Therefore the leakage between the two aquifers is strong and is responsible for the tide fluctuations observed in the unconfined aquifer (Figure 11a). Moreover, a vertical leakage exists also in an second area where the presence of the dam has no influence. Indeed, as indicated by the piezometric variations in Wells #3802 and #3803 (showing a trend in the data), there is vertical seepage in this area too because the level of the water in the semi-confined aquifer (≈ 7.25 m) is stronger than the unconfined aquifer (≈ 7.06 m) (Figure 10b). In this case, the vertical leakage does not create an increase of the tidal fluctuations of the unconfined aquifer (Figure 11b). Therefore, the

leakage between the two aquifers is related to the regional vertical flow is coming from a karstified chalk aquifer, which is not eliminated by the detrending of the piezometric data. This demonstrates that the effect of the vertical flow must be taken into account in our modeling and the classical approaches reported in the literature would not work in this case. Li et al. (2002) showed that the leakage of a semi-confined aquifer into an overlying unconfined aquifer damped significantly the tidal propagation. Several authors have used analytical models to examine such an effect on the estimation of the hydraulic diffusivity $D=T/S$ (Jiao and Tang, 1999, Li et al. 2001).

We estimate now the distribution of the vertical leakage coefficient L in the inverse modeling process. The inversion of the distribution of this parameter will be limited to the zones where the leakage is expected to be strong enough. These areas are discretized into several blocks (shown in Figure 13). We attempt to quantify the vertical infiltration coefficient L in each of these blocks where the leakage was observed at the piezometers. As explained above, we used zoning parameterization instead of the pilot points approach due to the difficulty of estimating an experimental variogram for the vertical infiltration coefficient (its distribution being disocntinuous). In our algorithm, the model vector of L-values is determined at the same time that we determine the transmissivity model vector at the position of the pilot points. The McMC approach is used to determine the hydraulic properties by generating, at each iteration, the transmissivity and the leakage models.

Geological, hydrogeological, and geotechnical information exist at a set of wells. They include (i) static penetration data (recording the stiffness and the type of the formations during drilling) and (ii) hydraulic pumping tests conducted at Wells #3794 and #3802. In addition, the analysis of the grain sizes D of the sediments recovered during the drilling of the boreholes allows to provide an estimate of the permeability (in m^{-2} below) through the relationship proposed by Shepherd, (1988). These combined information are used to construct the initial observed transmissivity field. This field is shown in Figure 12a. The initial distribution of the

transmissivity field is obtained by kriging the observed values of $Log_{10}T$ using the ordinary kriging approach. We obtain an experimental spherical isotropic variogram with a range of 3000 m and a sill of 1 $(Log_{10}[m^2s^{-1}])^2$ for kriging. The performance of the inversion modeling is heavily attributed to the perturbation field generated at the position of the 72 pilot points are reported on the Figure 12b. The forward problem operator $\boldsymbol{h}\ \boldsymbol{m}$ is obtained from the finite element solution of the transient Darcy equation with 46,080 triangular meshes (the mesh is shown in Figure 9) and one minute as time-step in the numerical modeling. We checked by trial and error that this mesh is fine enough to avoid numerical artifacts. The storage coefficient is assumed to be constant and known over the whole domain (the mean is $Log_{10}\ S$=-5 and the variance of 0.11) according to pumping test on the Wells #3794 and #3802). A total of 30,000 iterations were used to reconstruct the perturbation field and the leakage coefficients which required a computation time of 5 days with 2 cores. The perturbation field was provided from the kriging of the mean of the values of pilot points sampled. The Figure 14 represents an example of the hydraulic head signature produced at Well #3806 from a set of realizations 1000 sampled from the posteriori probability.

The spatial variability of the transmissivity field (see Figure12b) reveals that semi-confined aquifer is quite heterogeneous with transmissivity values ranging from 10^{-1} to 10^{-5} m^2 s^{-1}, so covering 4 orders of magnitude. We are now checking that the inverted field is consistent with the following independent observations:

(1) From the inversion, we observe an area characterized by low transmissivity values ($<10^{-4}$ m^2 s^{-1}) located on the foot of a cliff (called Zone A in Figure 12b). This area is consistent with the presence of poorly-sorted sediments including fine sands and silty clays according to the sedimentological analysis of core samples extracted from the wells present in this area (see Figure 15). Moreover, these silty and clayey sediments were also mentioned in Wells #3804 and

#3806 (called Zone B in Figure 12b). Therefore our inverse modeling is consistent with these observations.

(2) The high transmissivity values inverted ($>10^{-3}$ m^2 s^{-1}, see Zone B in Figure 12b) for the semi-confined aquifer are also consistent with the presence of alluvial deposits dominated by gravels and sands (see Figure 15).

The map of the standard deviations on the estimate of the transmissivity field provides an idea of the confidence interval of our result (shown in Figure 12c). Most of the uncertainty on the transmissivities are lower than 0.2 $\text{Log}_{10}(T$, in m^2 s^{-1}), a quite narrow range. Therefore the inverted transmissivity distribution is quiet thrustable and the Root Mean Square between the inverted and observed data is 3% (data RMS error), which means that the hydraulic head data are very well reproduced by our most likely permeability and leakage coefficient models. However, we note that the head response in some wells (especially in Wells#3804 and #3798) is not reproduced. At these locations, we are faced with a complex (likely non-linear) behavior due to a memory effect of the tidal fluctuations of the heads is the result of the effect of the tide variations of the several previously hours or days. Such a cumulative effect can be observed in areas where groundwater levels do not react immediately to current changes of the tide due to the low transmissivity or to the large distance that separates between the river from this area. At the opposite, in the case of Wells #3804 and #3798, the difference in the computed and observed heads is smaller than 2 cm. This is an excellent result especially if we consider the total head variations (>75 cm, see Well #3812 for instance).

In the area dominated by the vertical seepage, the leakage coefficient L is quite strong ($L \approx 10^{-1}$ day^{-1} in the vicinity of the Wells #3802 and #3804). This is consistent with the fact that in this area, the silty-peat aquitard deposit is absent or very thin as shown by drilling (#3810). On the other hand, the low values of the leakage coefficient around Wells #3522 and #3808 are consistent with the head pressures recorded in both aquifers (Figure 11c). The presence of the dam is also responsible for the spatial variability of the vertical leakage. High values of L ($\approx 10^{-1}$

day^{-1}) have been estimated in the area where the size of channel is reduced and the cliff shown in Figure 7 is close to the river. We note that the estimate of the standard error for the vertical leakage coefficient varies between 0.8 and 1.1 $Log_{10}(L$, in s$^{-1})$.

The parameters of the variogram retrieved through the inversion process are 2730 m for the range and 0.84($Log_{10}[m^2s^{-1}])^2$ for the sill, these parameters are consistent with the parameters of experimental variogram.

5. Conclusion

The hydraulic head of alluvial heterogeneous aquifers oscillates in response to the tidal variations of the level of a river in its estuary. This hydraulic response can be analyzed in the framework of linear poroelasticity. We have developed a new approach to map both the spatial distribution of the transmissivity field (assuming the storage coefficient constant) and the vertical leakage coefficient between a semi-confined aquifer and an unconfined aquifer. The tidal variations recorded in the river estuary are used as a boundary condition in order to solve the flow (diffusion) equation. The inverse problem consists in retrieving the heterogenous transmissivity field from the head observations. This inverse problem is non-linear and therefore likely to be non-unique. We solve this inverse problem by generating an initial kriged transmissivity field from the measured local values of the transmissivity at a set of selected positions. This initial transmissivity field is perturbed to fit the measured heads with a set of complementary transmissivity data at 72 pilot points. The position of these pilot points are uniformly distributed within the aquifer on a regular grid. The optimal perturbation field created thanks to the transmissivity values at the pilot points are determined using an McMC algorithm. This approach has been tested successfully on a synthetic case study and was used to invert the transmissivity field of a semi-confined coastal aquifer at a test site located in the West of France in the vicinity of the estuary of the Seine River. The framework developed in our work can be

used to incorporate additional information of geophysical nature. We plan to include static and time-lapse geophysical information to estimate the transmissivity, the specific storage, and the leaking coefficient distributions.

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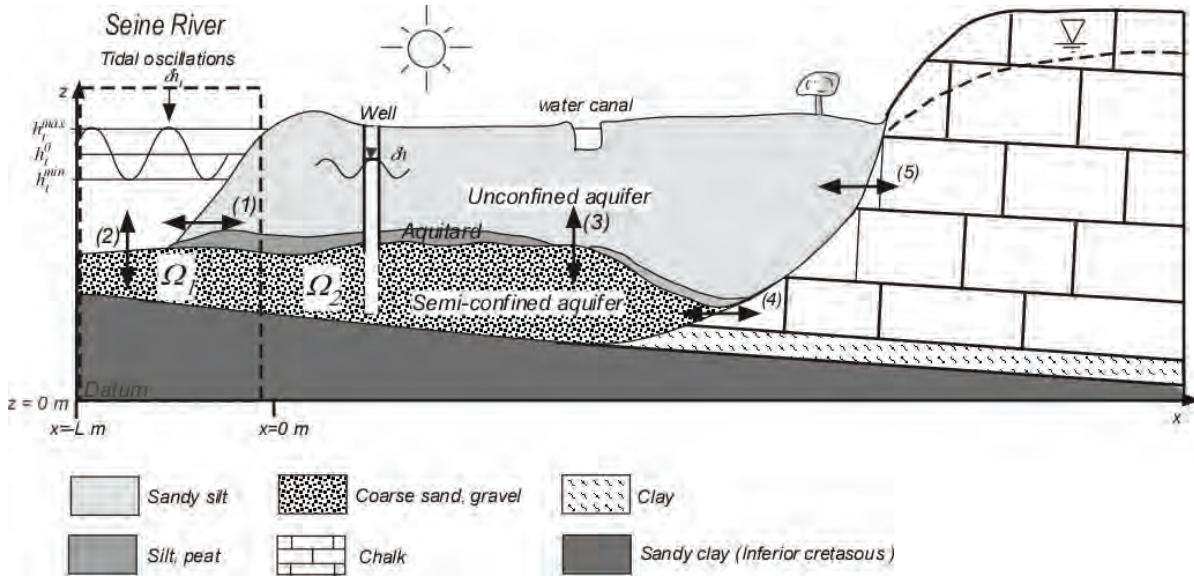
Figure 1. Sketch of an alluvial aquifer under a tidal influence of a river in its estuary. The hydraulic system consists of two aquifers, a shallow one that is unconfined and one semi-confined aquifer. The domain Ω_1 represents the area for which both aquifers are covered by the river. The second domain Ω_2 represents the inland part of the (semi-)confined and unconfined aquifers. The semi-confined aquifer is in interaction with Seine river (see exchange 2), with the unconfined aquifer (see exchange 3), and the karst aquifer in the carbonate (chalk) formation (see exchange 4). The unconfined aquifer is interacting with the karst aquifer and the Seine River (1 and 5).
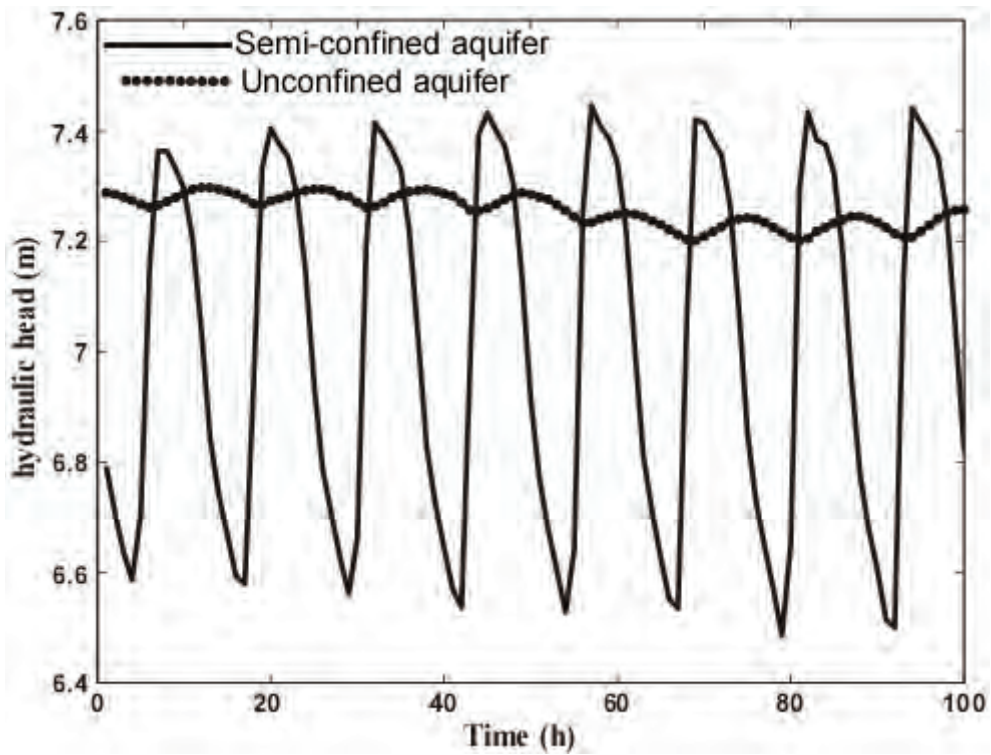
Figure 2. The hydrographs recorded in the semi-confined and unconfined aquifers show that the tidal head fluctuations in the unconfined aquifer are negligible compared to the head fluctuations in the semi-confined aquifer.
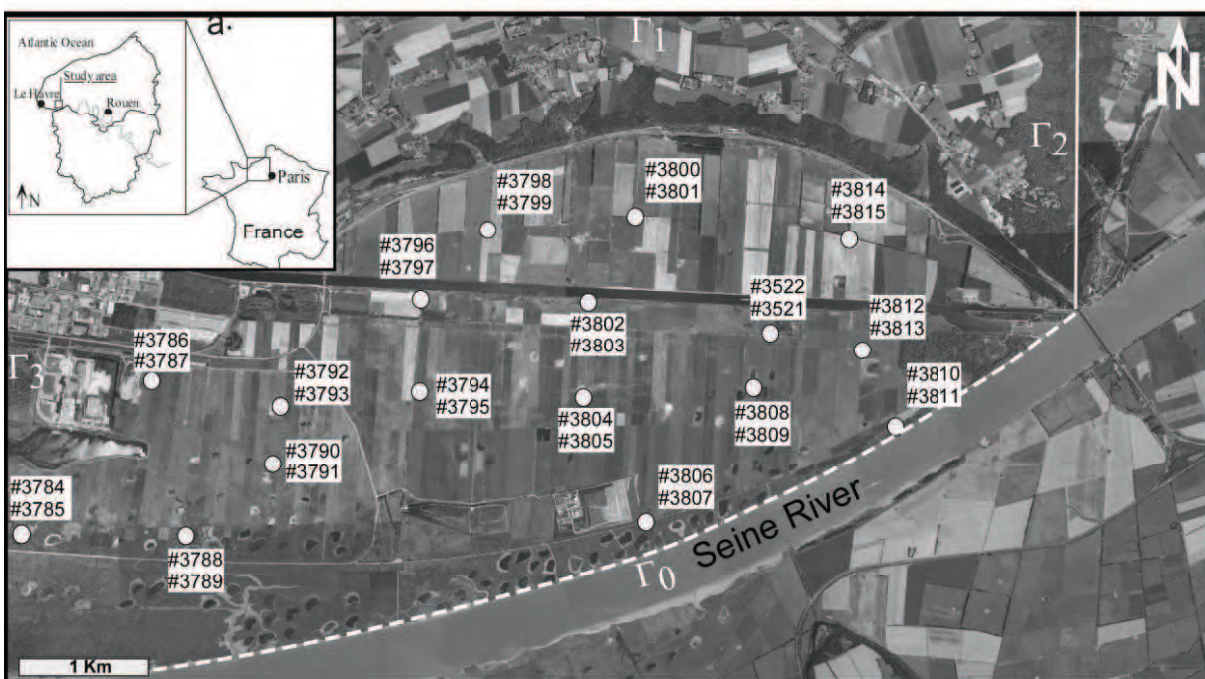
Figure 3. Map of the test site. **a.** Our test site is located in Upper Normandy (France). The south boundary of the field corresponds to the position of the estuary of the Seine River. **b.** This site benefits from the presence of 17 set of piezometers (each set comprises several well drilled at different depths to study both the semi-confined and unconfined aquifers). The piezometers located in the semi-confined aquifer have even numbers for their name and we use odd numbers for the piezometers located in the unconfined aquifer. The parameters Γ_0, Γ_1, Γ_2 and Γ_3 denote the boundaries of the study area.
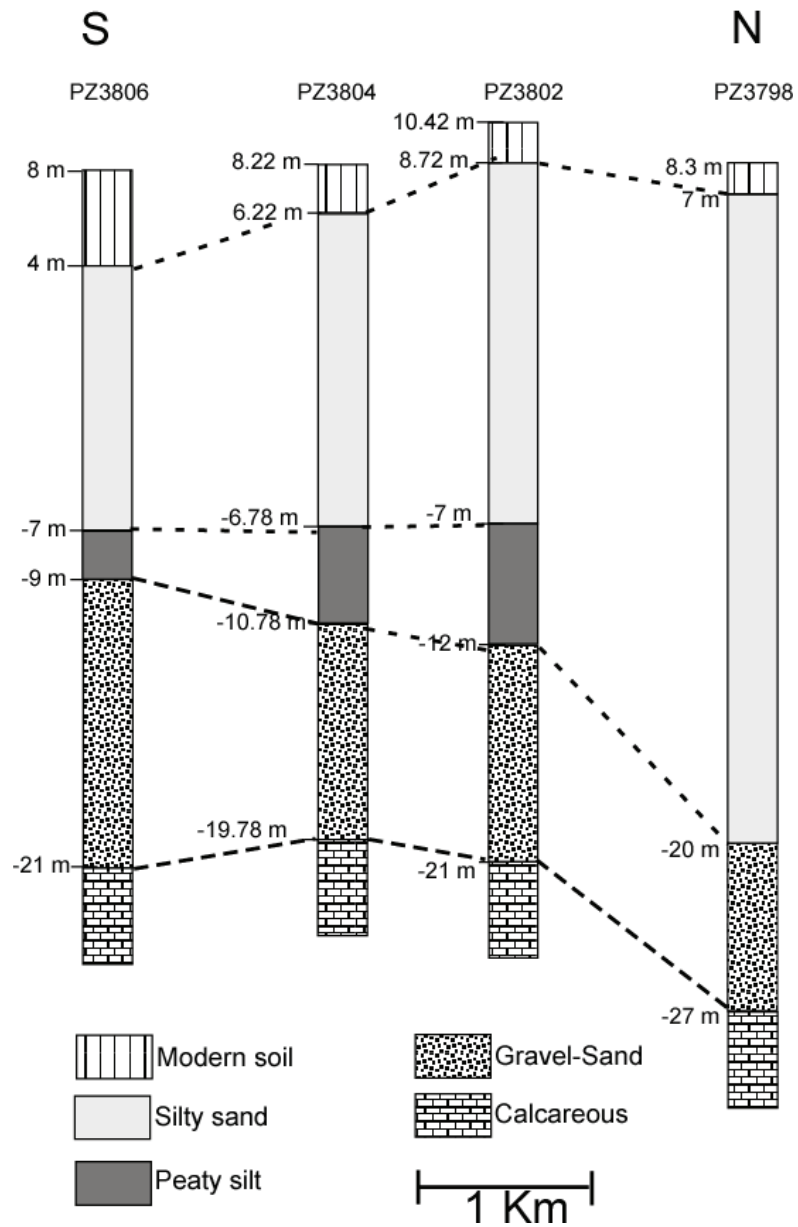
Figure 4. North to West geological cross-section of the test site. The fluvial deposits are composed by coarse sand and gravel in the bottom and silty sand deposits in the top. The two deposits are separated by a discontinuous silty peat aquitard.
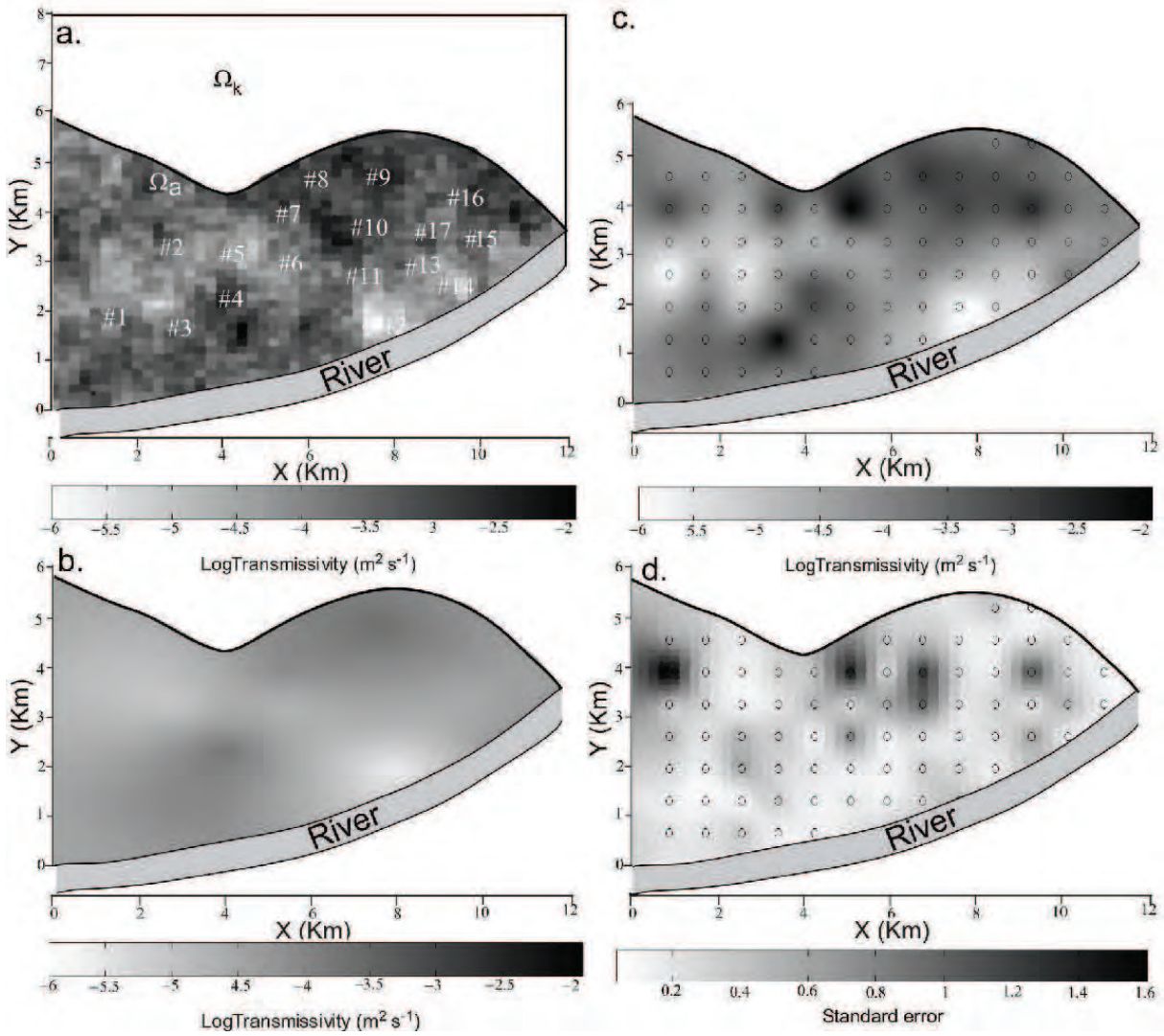
Figure 5. Numerical synthetic test. **a**. Because of the boundary conditions, the field study includes a domain in contact with the chalk aquifer Ω_k in which a homogeneous transmissivity is assumed. For the alluvial domain Ω_a, the Log-transmissivity field of the hypothetical aquifer model is generated by the SGSIM geostatistic code. The semi-confined aquifer is submitted to tidal river forcing in its estuary. A set of wells is randomly localized in the domain (see the symbol #). **b**. The transmissivity field is obtained using ordinary kriging of the $\log_{10}T$ of the five measurements. **c**. We use 72 pilot points corresponding to the circles. The transmissivity are inverted at these pilot points using the tide response recorded at the 17 wells. **d**. Standard error map of the inverted transmissivity field.
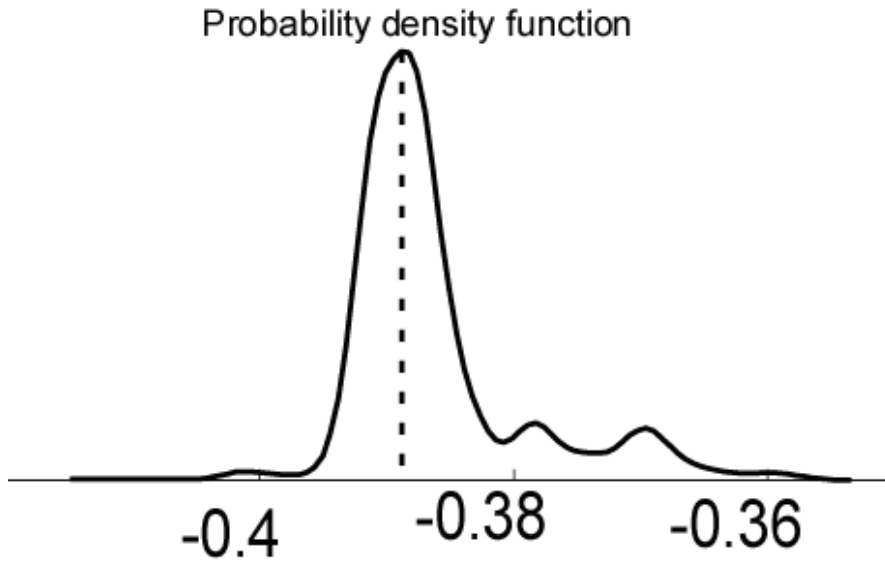
Figure 6. An example of the posterior probability density function normalized of the sampled models of the pilot point where the mean and variance were derived. This distribution represents the last 5000 iterations of pilot point for which the McMC sampler has converged. The bar indicates the mean value of the distribution used to reconstruct inverted transmissivity filed.

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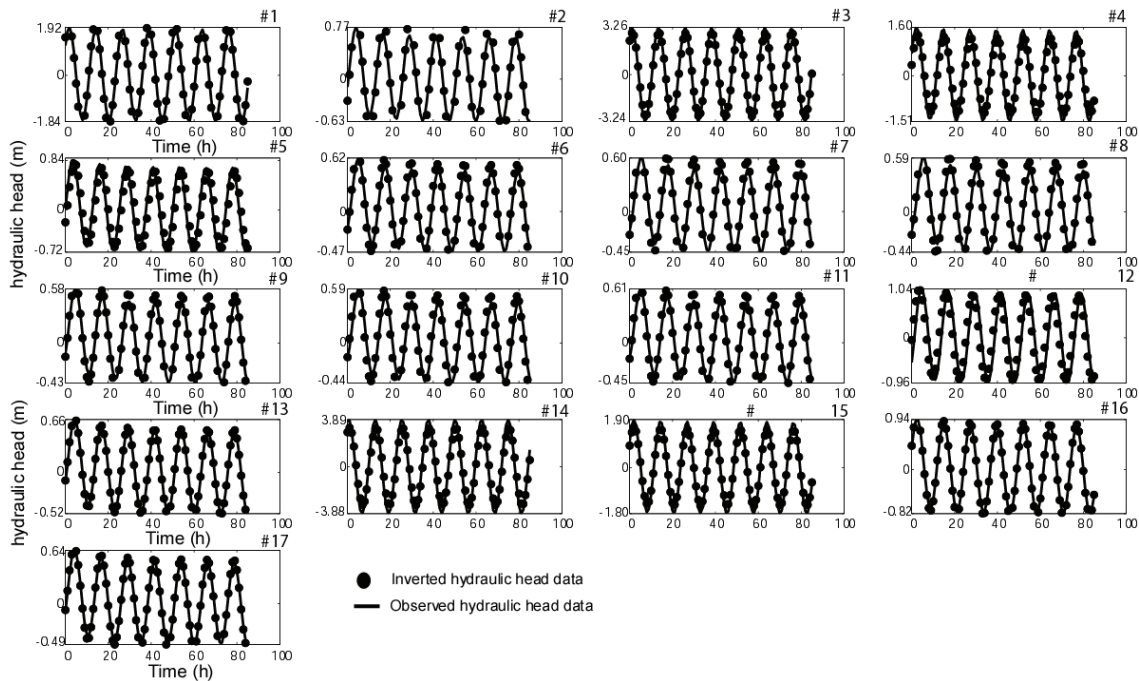
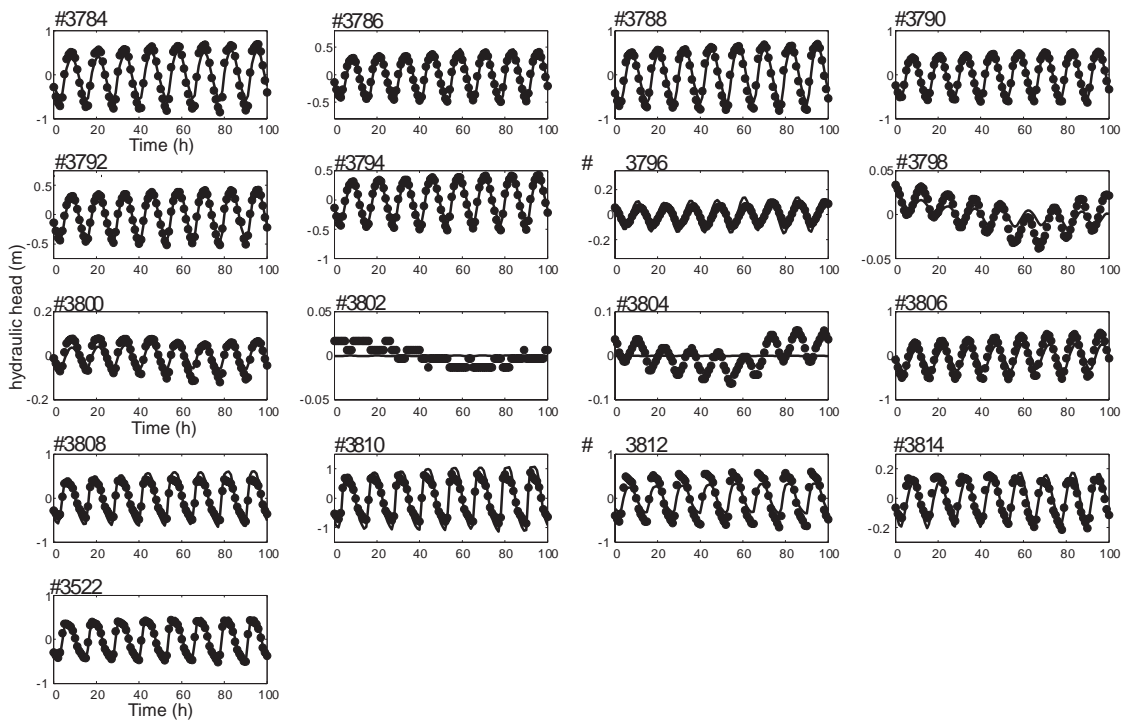**Figure 7.** Comparison between the observed and inverted well hydrographs in the semi-confined aquifer.
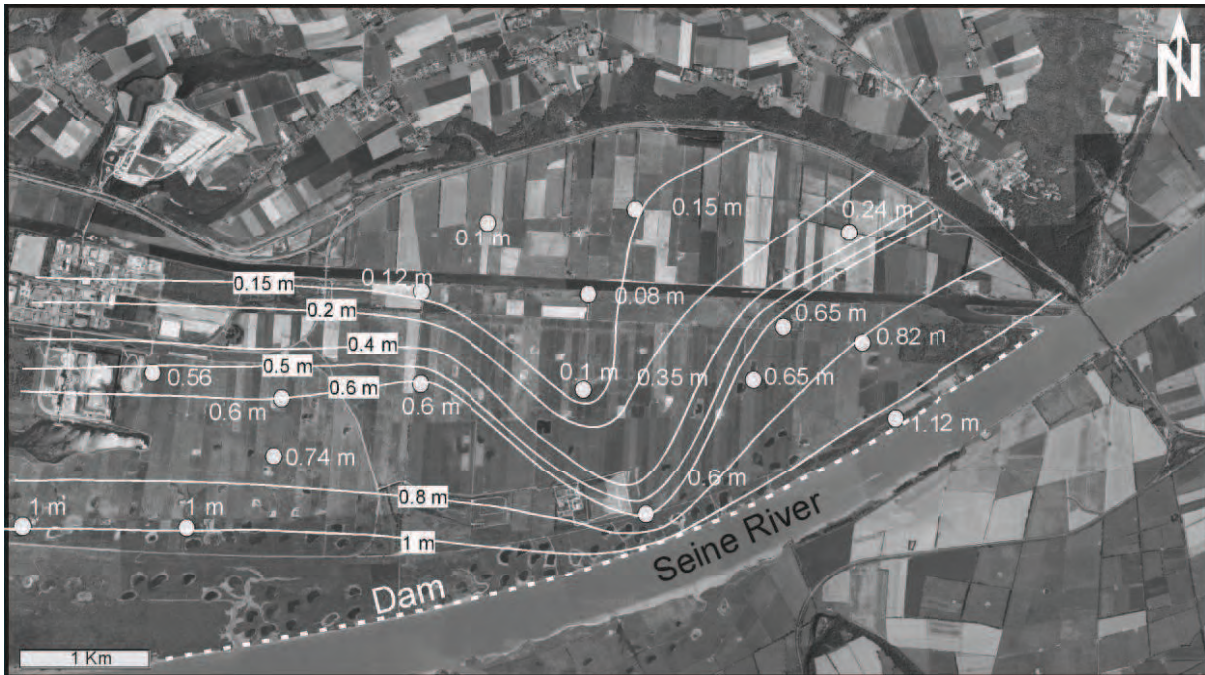
Figure 8. Map of the amplitude of tidal fluctuations showing the dependence of the amplitude to the distance to the river as well as the dependence to the transport properties (transmissivity and specific storage) of the aquifer and the vertical leakage from the semi-confined into the unconfined aquifer. The dam is schematized by dotted line. The base of dam is built on the surface deposits on the river bank.
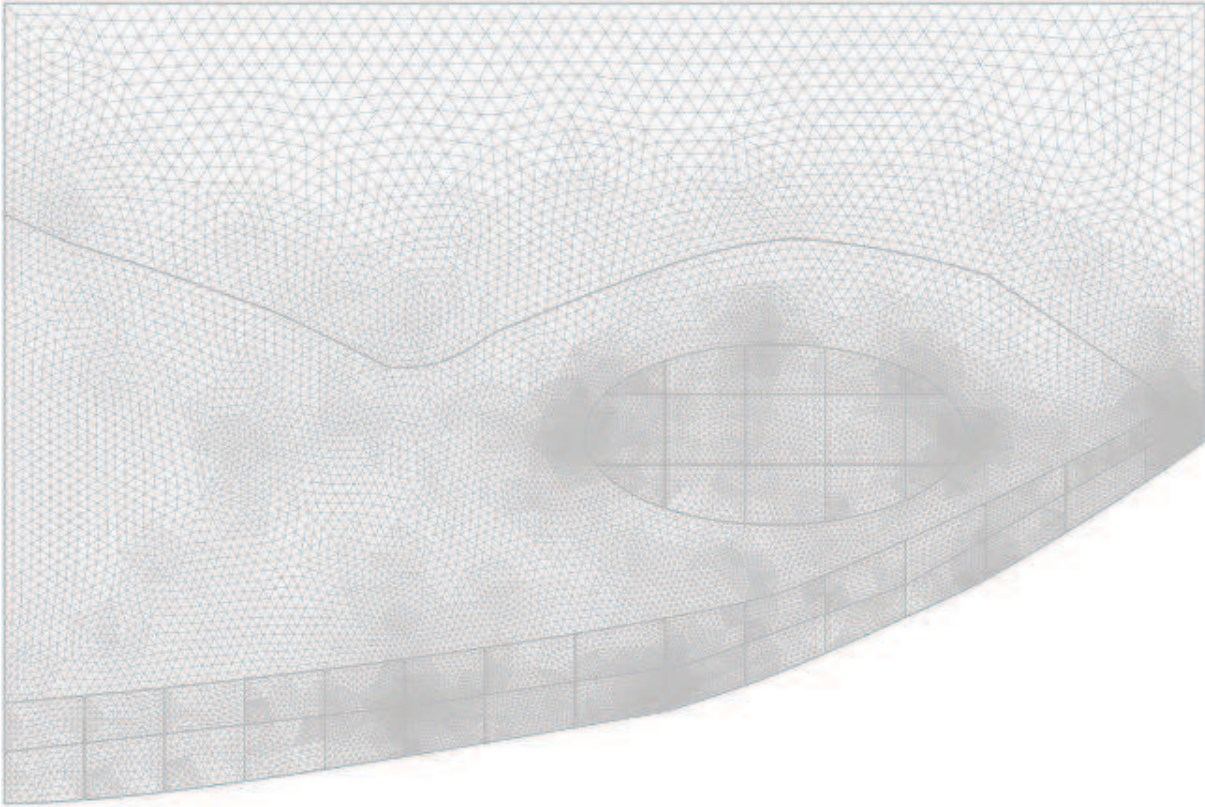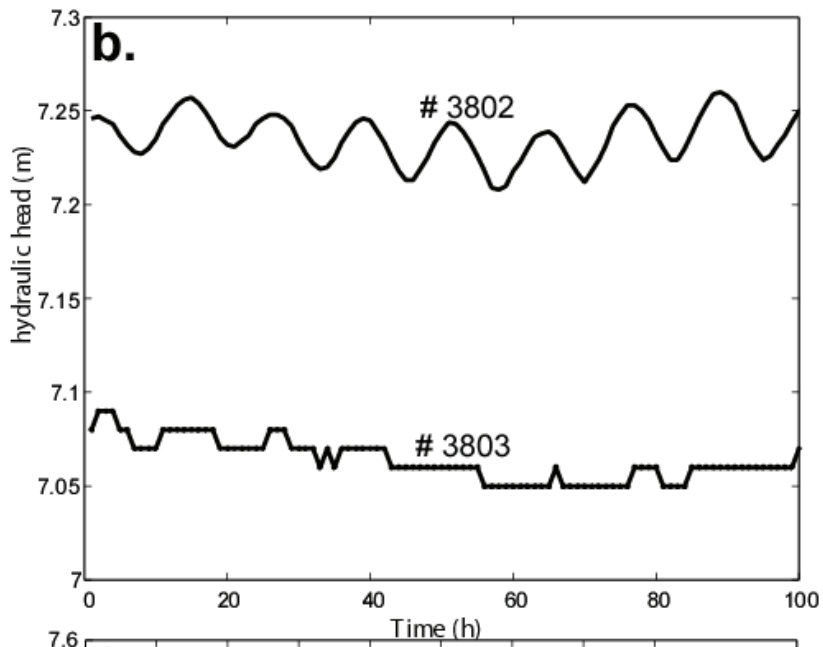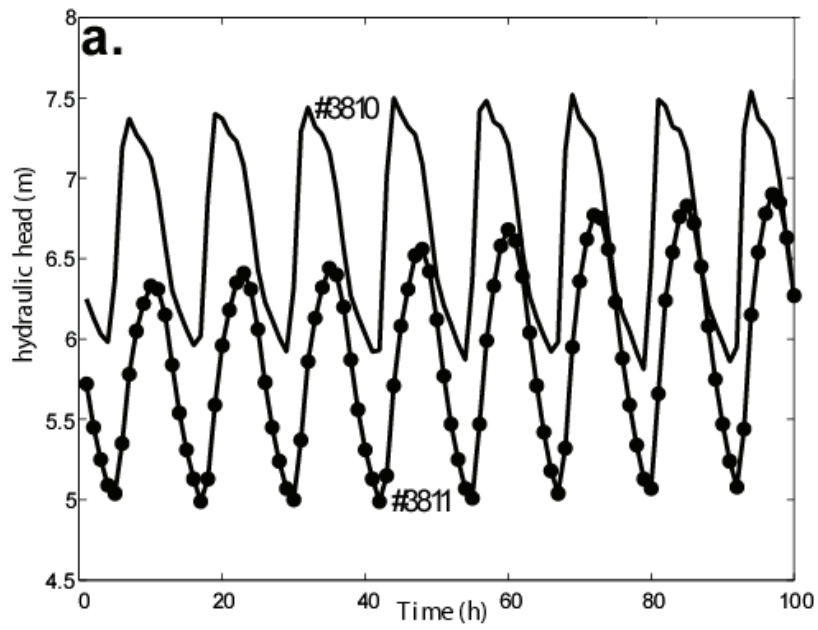
Figure 9. The triangular mesh (46,080 triangular meshes) used in the finite element technique to solve numerically Darcy equation in the transient state.

Figure 10. Comparison between the observed (filled circles) and inverted (plain lines) well hydrographs in the semi confined aquifer.
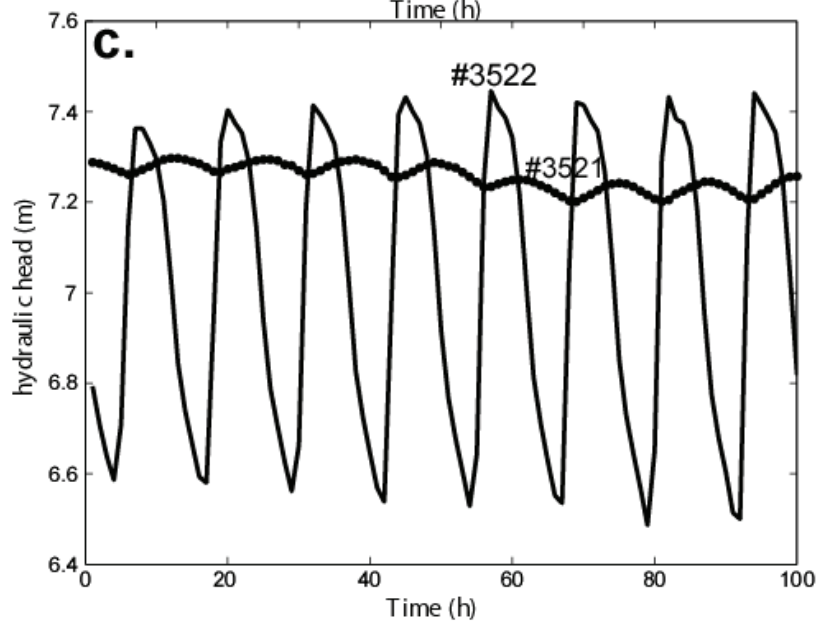
Figure 11. Some hydrographs recorded in the semi-confined and unconfined aquifers in three distinct zones associated with the influence of the vertical leakage from the semi-confined aquifer into the unconfined aquifer. The displayed hydraulic data presented are not corrected so they include a time-dependent component associated with regional flow. In these three areas, the hydraulic head of the unconfined aquifer becomes lower compared to the semi-confined aquifer. Wells #3810 and #3810 show the importance of the vertical flux associated with the presence of the dam. Wells #3802 and #3803 show the hydraulic head behavior of the area where the vertical flux is associated with the regional groundwater flow. There is no hydraulic head variations in Well #3803 and so this area is insensitive to tidal effects. Wells # 3522 and # 3522 correspond to an area where the vertical infiltration is present transiently only during high tides.
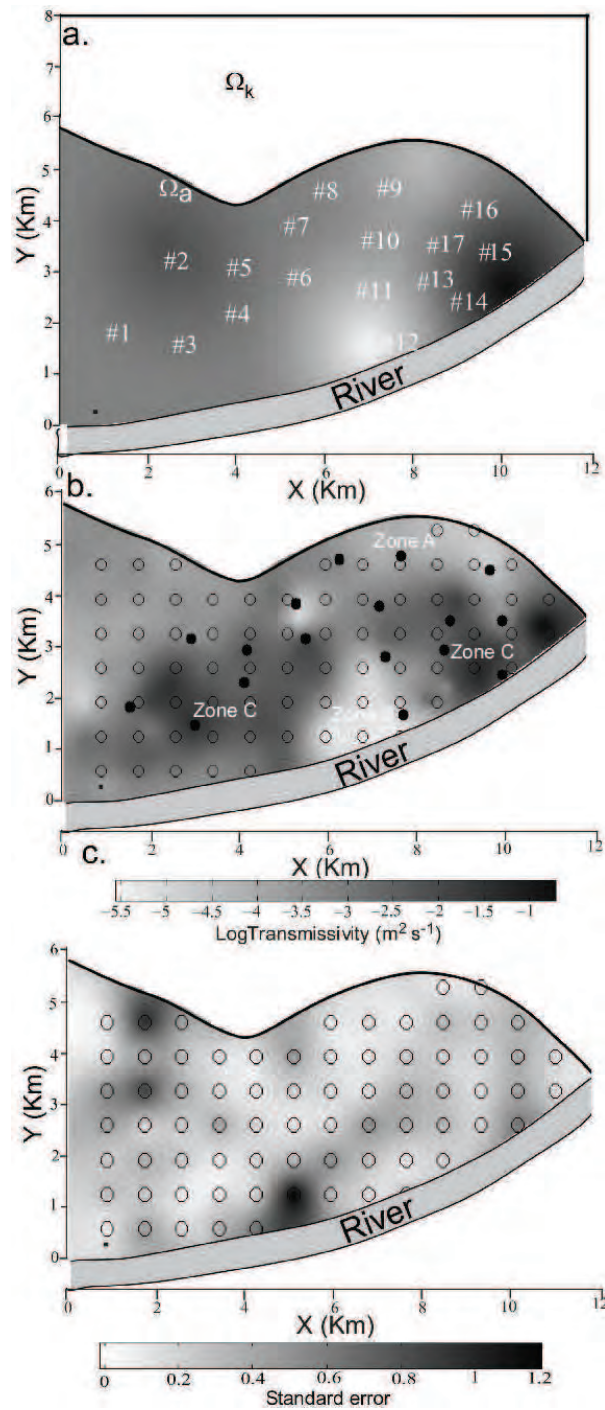
Figure 12. Inverted transmissivity field. **a.** The Log- transmissivity field using ordinary kriging of the $\log_{10}T$ measurements. **b.** Position of the 72 pilot points (open circles) and the piezometers (black circles). The values of the transmissivity are inverted at the pilot points using the tide response recorded at the 17 wells shown in Figure 3b. The map shows two areas of low transimissivity (zones A and B) and one area of high transmissivity (zone C). **c.** Standard error distribution of the transmissivity field.
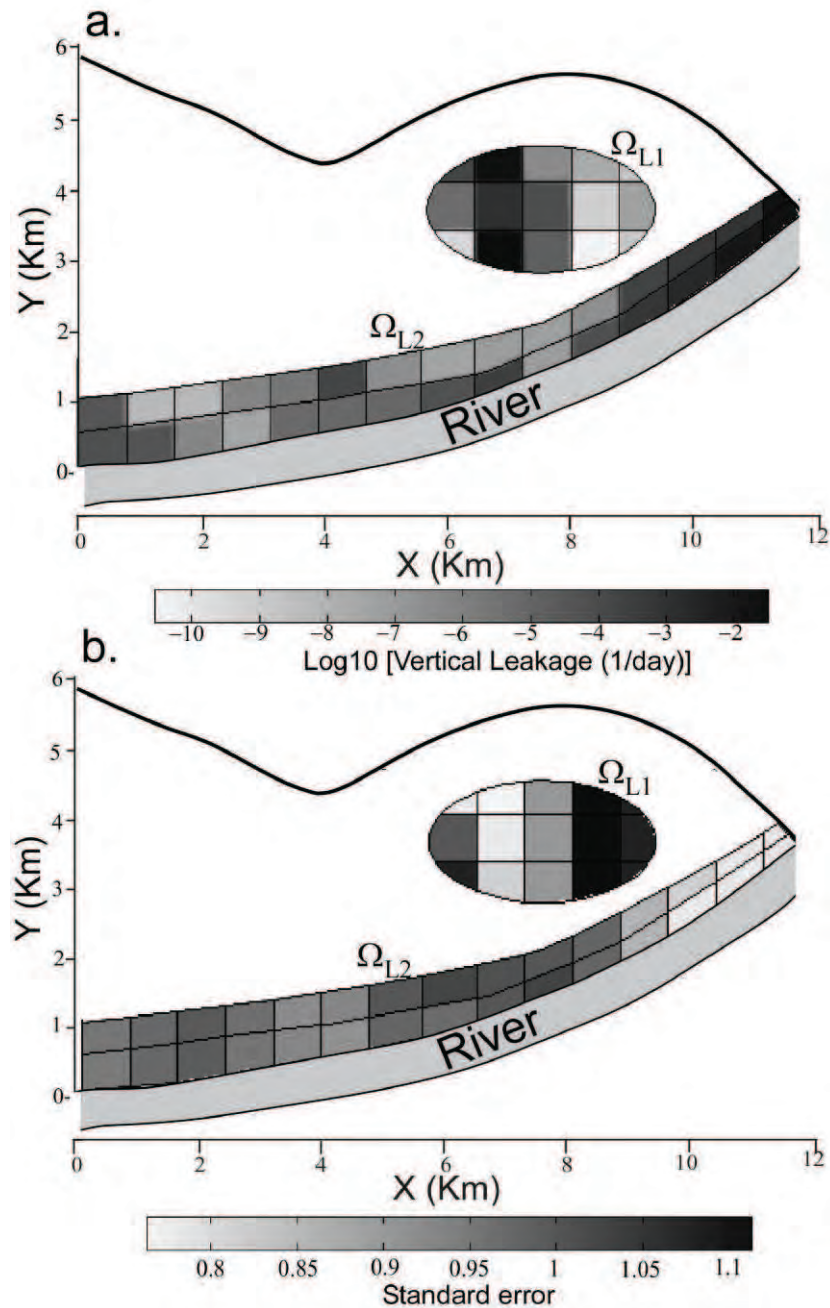
Figure.13. Estimate of the distribution of the vertical leakage coefficient *L* from the semi confined into the unconfined aquifers. The black dots correspond to the pilot points positions 934 used for the vertical leakage calibration. The areas Ω_{L2} and Ω_{L1} correspond to areas where vertical leakages are observed. The area Ω_{L1} corresponds to an area where the vertical leakage is likely related to the regional flow. The area Ω_{L2} corresponds to an area where the vertical leakage may result from the presence of the dam.
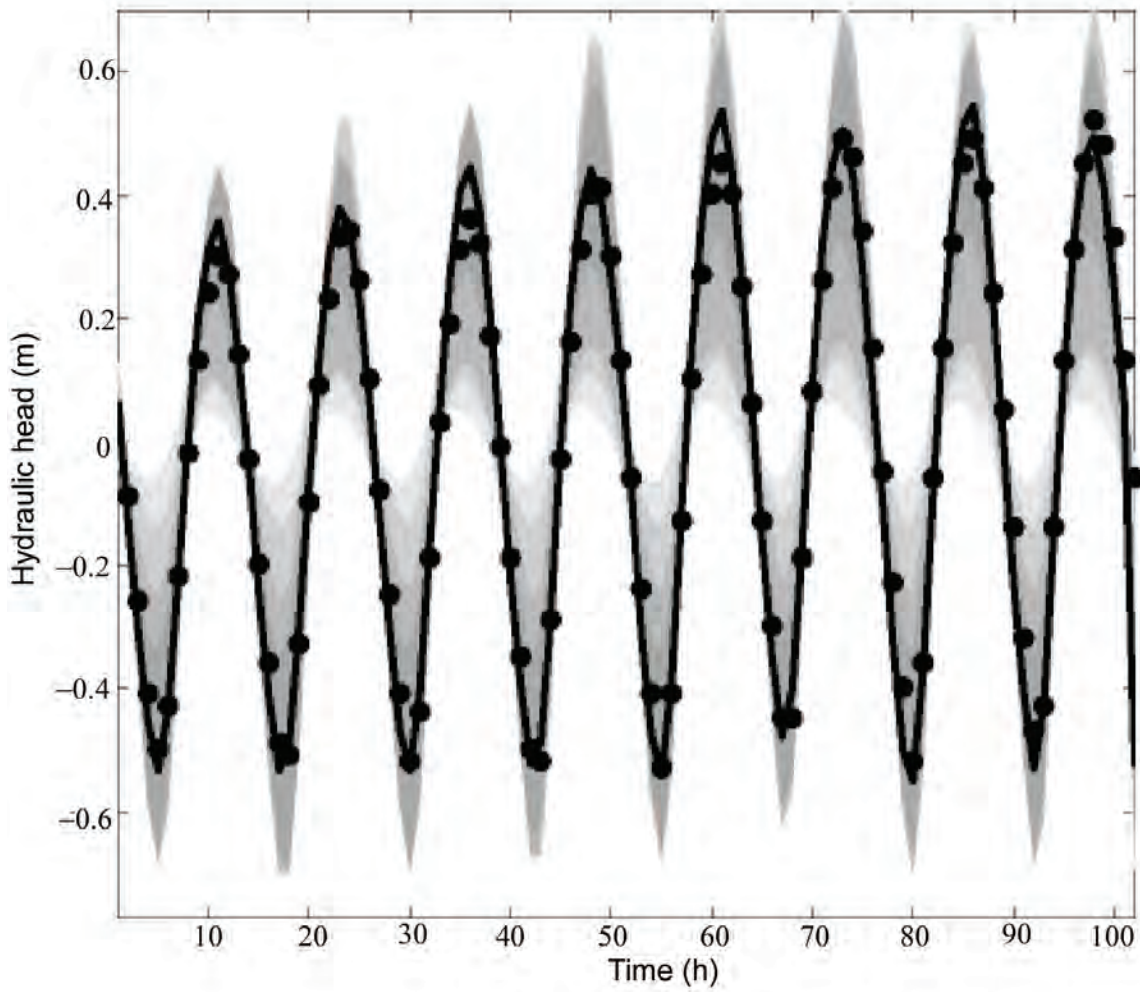
Figure 14. Predicted hydraulic head of Well # 3806 obtained using different sets of realizations (1000). The solid lines corresponds the mean predicted curves. The filled circles show the observed data.
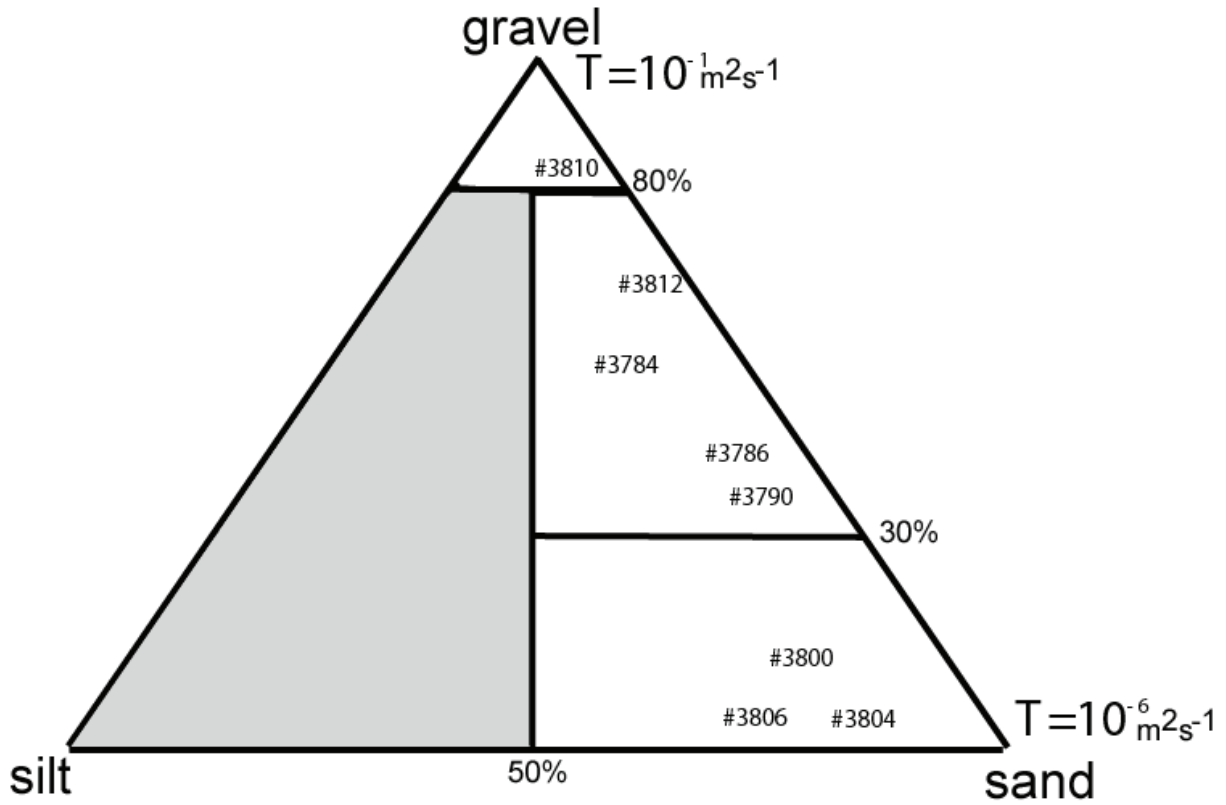
Figure 15. Textural categories of the semi-confined aquifer derived from the analysis of core recovery from available wells. This analysis reveals that the hydraulic transmissivity values estimated in concordance to the texture of the porous material.

ANNEXE B

Stochastic joint inversion of the geophysical cross-hole electrical data for salt tracer test to image hydraulic conductivity

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Abstract. The assessment of hydraulic conductivity of heterogeneous aquifers is a difficult problem to solve using traditional hydrogeological methods (e.g., steady state or transient pumping tests) due to their low spatial resolution. Geophysical measurements performed at the ground surface and in boreholes provide additional information for increasing the resolution and accuracy of the inverted hydraulic conductivity. We use a stochastic joint inversion of Direct Current (DC) Electrical Resistivity Imaging (ERI), self potential tomography (SPT), plus in situ measurement of the salinity collected in time during a synthetic salt tracer experiment to reconstruct the hydraulic conductivity field. The pilot points parameterization is used to avoid over-parameterization of the inverse problem. The bound- constraints are included in the Markov chain Monte Carlo for promote a consistent sampling with the hydrogeological conditions. To evaluate the effectiveness of the inversion process, we compared several scenarios where the geophysical data are coupled or not to the hydrogeological data to map the hydraulic conductivity field. We first examined the effectiveness of the inversion of each type of data, and then we proceeded to combine the methods two by two. We also combined all the information together to show the value of each type of geophysical data in the inversion process. The results of the inversion revealed that the self-potential data improved the estimate of hydraulic conductivity especially when the signature of self-potential is combined to the salt concentration measurement in the second well or to the time-lapse electrical resistivity data. The best strategy remains the time-lapse joint inversion of all the data (self-potential, resistivity, and salinity measurements in the second well).

1. Introduction

The steady-state flow of the ground water and solute transport are mainly controlled by the spatial distribution of the permeability and dispersivity of an aquifer. Permeability can vary over 12 orders of magnitudes and can extremely heterogeneous at various scales, which in turn implies a complex pattern for groundwater flow and contaminant transports [1]. The limited number of available piezometer makes the reconstruction of the hydraulic conductivity of heterogeneous aquifers a difficult problem if only pumping tests are performed [2, 3]. The use of geophysical methods can provide additional complementary information useable to invert the hydraulic conductivity field [4, 5].

Recently, geophysical tools have benefited from (i) the evolution of the efficiency of numerical methods (e.g., the mixed finite element) for solving partial differential equations and parallel computing [e.g., 6,7,8], (ii) the development of petrophysical models connecting the geophysical signature to the hydraulic properties [e.g., 9,10,11], and (iii) significant improvements in the technology of various sensors and filtering techniques. These developments have therefore given birth to a new era of three-dimensional time lapse geophysical imaging for tracking the changes of the variables like the moisture content, the salinity, and the pore fluid pressure [12].

Along these lines, electrical resistivity imaging ERI has been used for mapping the hydraulic conductivity field using salt tracer experiments [13]. To evaluate the hydraulic conductivity field, Linde et al. [14] used salt tracer tests with cross-well GPR. Hyndman et al. [15] presented a petrophysical relationship connecting the seismic slowness and hydraulic conductivity, which has been successfully used to predict the hydraulic conductivity of an alluvial aquifer from seismic and tracer test data. Other geophysical methods include the joint inversion of self-potential and temperature measurements (both being passive methods that are

directly sensitive to the flow of the pore water [16]), induced polarization measurements [17], and magnetic resonance sounding [18].

To reduce the uncertainties in the estimation of the hydraulic conductivity, hydrogeophysical and hydrogeological information need to be somehow coupled. Two approaches can be used for that purpose. In the first approach, the hydrological information is used to constraint the inversion of the geophysical measurements [14]. The second approach is to fully coupled the inversion of the two types of data [4, 19, 20]. Hinnells et al. [21] devoted a study on the advantages of coupling hydrological and geophysical data to reconstruct the hydrological parameters using ERI for tracking the infiltration front in vadose zone in a synthetic case study.

In this paper, we are interested by the joint inversion of geophysical data to monitor salt tracer test and to assess the hydraulic conductivity field. Inverting salt tracer tests using geophysical method has been investigated by many researchers [22, 23, 24, 25, 26, 27]. However, the non-uniqueness of the ERI inverse problem (and its sensitivity map) makes this method insufficient by itself and requires that ERI needs to be combined with other sources of information like in situ measurements of the salt concentration in wells [see 23, 15, 28]. Irving and Singha [15] introduced a stochastic joint inversion approach of time-lapse cross-well ERI and salt tracer concentration data. Our work is following this idea but adding an additional method, the self-potential method, to the inverse problem. Self-potential (SP) measurements are passive electrical potential measurements associated with the occurrence of a natural electrical current associated with the flow of the pore water itself. This current is also affected by the salt concentration as discussed in the literature [29]. We use the pilot points approach for the joint inversion of ERI, SP, and salt tracer data because this method reduces the over parametrization of the problem. While previous authors used deterministic methods to choose the values of the pilot points, we place the pilot points on a regular grid and we use a stochastic method based on

the Markov chain Monte Carlo, McMC, approach to estimate the values of the model parameters at the pilot points.

2. Theory and Methods

2.1. Governing Equations

We present first the equations governing the physical processes of groundwater flow and saline tracer transport in a heterogeneous unconfined aquifer. We also introduce the semi-coupled equations connecting the salt concentration to the electrical resistivity (to interpret ERI) and to the source current density used to interpret SPT.

a. 2D Flow and Transport Equations

In the steady-state condition, the governing groundwater flow equation in saturated and heterogeneous porous media can be expressed in 2D by:

$$\nabla \cdot (K\nabla h) = 0, \tag{1}$$

subject to the boundary conditions

$$h = h_0 \text{ at } \Gamma_D, \tag{2}$$

$$-\boldsymbol{n}.K\nabla h = q_0 \text{ at } \Gamma_N, \tag{3}$$

where h is hydraulic head (in m), K is the hydraulic conductivity (m s^{-1}, we assume that each subelement of the aquifer is isotropic). Equations (2) and (3) correspond to Dirichlet and Neumann boundary conditions, respectively. The hydraulic head h_0 denote the head fixed at the boundary Γ_D, q_0 is the hydraulic flux (m^2 s^{-1}) assumed at the Neumann boundary Γ_N, and $\boldsymbol{n}$ is the unit vector normal to the boundary Γ_N.

The constitution of the transport equation of a salt tracer consists of the coupling of the Darcy's law for Darcy velocity u (in m s^{-1}) and Fick's law for the flux of the salt $\mathbf{j}_d$ (in kg m^{-2} s^{-1}) [30]:

$$\mathbf{u} = -K\nabla h, \tag{4}$$

$$\mathbf{j}_d = -\rho_f \phi \mathbf{D} \cdot \nabla c + \rho_f \mathbf{u} c, \tag{5}$$

where $\mathbf{u}$ denotes the Darcy velocity (m s^{-1}), $\mathbf{D}$ (in m^{2}s^{-1}) denotes the hydrodynamic dispersion tensor, ϕ (unitless) denotes the connected porosity, c denotes the solute mass fraction (unitless), and ρ_f represents the solute bulk density (in kg m^{-3}).

The transport of the salt due to the injection of a salt tracer in the aquifer follows the advection-dispersion equation derived from a combination of the continuity equation (mass conservation equation for the salt) and a generalized Fick's law:

$$\frac{\partial(\rho_f \phi c)}{\partial t} + \nabla \cdot \mathbf{j}_d = 0. \tag{6}$$

Eq. (6) can be rewritten under the following partial differential equation:

$$\phi \frac{\partial c}{\partial t} + \nabla \cdot \left[-\phi \mathbf{D} \cdot \nabla c \right] + \mathbf{u} \nabla c = 0, \tag{7}$$

which is subject to the following initial and boundary conditions:

$$c = 0, \forall \mathbf{x} \text{ at } t = 0, \tag{8}$$

$$c(\mathbf{x}, t) = c_0 \text{ at } \Gamma_1, \tag{9}$$

$$-\mathbf{n}. \mathbf{D}\nabla c = 0 \text{ at } \Gamma_2. \tag{10}$$

In these equations, c_0 (unitless) denotes the solute mass fraction of the salt in the source term. In the Fickian model, the hydrodynamic dispersion tensor is given by

$$\mathbf{D} = \left[\frac{D_m}{\alpha} + \alpha_T v \right] \mathbf{I}_3 + \frac{\alpha_L - \alpha_T}{v} \mathbf{v} \otimes \mathbf{v}, \tag{11}$$

where D_m is the molecular (mutual) diffusion coefficient of the salt (in $m^2\ s^{-1}$) (for a NaCl

solution, D_m is $1.60 \times 10^{-9}\ m^2\ s^{-1}$ at infinite dilution and is $1.44 \times 10^{-9}\ m^2\ s^{-1}$ at high salinities at

25°C), $v = |\mathbf{v}|$, $\mathbf{a} \otimes \mathbf{b}$ represents the tensorial product between vectors $\mathbf{a}$ and $\mathbf{b}$, and α_L and α_T

are the longitudinal (along $\mathbf{v}$) and transverse (normal to $\mathbf{v}$) dispersivities (in m), and α denotes

the tortuosity of the pore space. This bulk pore space tortuosity can be obtained as the product of

the electrical formation factor F entering into the electrical conductivity problem (see below) and

the porosity ϕ [31].

The numerical solution of the hydrodynamic-transport problem is performed by using

Comsol software. In the first step, we determine the distribution of the Darcy velocity by solving

Darcy's equation in quasi-static conditions, and then we use the Darcy velocity to solve the

advection-dispersion equation to determine the distribution of the salinity in space and time.

b. 2.5D Electrical Resistivity Imaging ERI

ERI consists in inverting apparent resistivity data obtained by injecting an electrical

current with two (current) electrodes and measuring the difference in the electrical potential at a

set of (voltage) electrodes (at least two electrodes). For an isotropic and heterogeneous

electrical conductivity distribution in 2D can be defined with the potential spectrum

$\tilde{V}(x, k_y, z)$ described as [32]:

$$\nabla \cdot \sigma \nabla \tilde{V} - k_y^2 \sigma \tilde{V} = -\frac{I_s}{2} \left[\delta \left(\mathbf{x} - \mathbf{x}_s^+ \right) - \delta \left(\mathbf{x} - \mathbf{x}_s^- \right) \right], \qquad (12)$$

with the following boundary conditions:

$$\tilde{V} = 0 \ \text{ at } \Gamma_d, \ \|r\| \to \infty, \qquad \textbf{(13)}$$

$$-\mathbf{n}.\sigma \nabla \tilde{V} = 0 \ \text{ at } \Gamma_N, \qquad (14)$$

where $\tilde{V}$ denotes the electrical potential (in V) due to dipole source modeled in 3D, σ denotes

the electrical conductivity (in $S\ m^{-1}$), $\mathbf{x}$ denotes the position vector of the source of current, I_s

corresponds to the injected current (in A), $\mathbf{x}_s^+$ and $\mathbf{x}_s^-$ are the locations of the positive and negative current sources, respectively, and k_y denotes the real wave number is determined by the algorithm discussed by Pidlisecky and Knight [32]. The boundary conditions are subdivided as a Neumann boundary condition Γ_N imposed at the insulting air-ground interface and a Dirichlet boundary condition Γ_d at the others boundaries [33].

The electrical resistivity problem (Equation 12 to 14) is solved numerically with the finite element approach for a set of the wave numbers k_y [32]. The solution $\tilde{V}$ is transformed from the wave number domain to the spatial domain following the approach of Dey and Morrison [33] using an inverse cosine-transform.

$$V = \frac{2}{\pi} \int_0^\infty \tilde{V} \cos(k_y y) dk_y \,. \tag{15}$$

The electrical conductivity of the soil is connected to the solute electrical conductivity according to the model of Waxman and Smits [34]:

$$\sigma = \frac{\sigma_f}{F} + \sigma_s \,, \tag{16}$$

where $F = \phi^{-m}$ (unitless) denote the electrical formation factor, σ_f denote the pore water electrical conductivity (S m^{-1}), and the power law exponent m (>1, unitless) denote the cementation exponent, which ranges from 1.3 to 1.5 for clay-free unconsolidated sands and from 1.8 to 2.0 for consolidated sandstones [10].

The pore water conductivity is related to the concentration c by a simple linear equation:

$$\sigma_f = c \left(\beta_{(+)} + \beta_{(-)} \right) e \,, \tag{17}$$

where c is salinity concentration of the pore water expressed in kg/m^3, e is the elementary charge of the electron (e=1.6 x 10^{-19} C), $\beta_{(+)}$ (25°C) = 5x10^{-8} m^2 s^{-1} V^{-1}), and $\beta_{(-)}$ (25°C) =7x10^{-8} m^2 s^{-1} V^{-1} denotes the mobility of the cations and anions (NaCl) in the pore water, respectively. From

Eqs. (16) to (18), the macroscopic electrical conductivity measurements are therefore related to the salinity by:

$$\sigma = c\phi^m e \left(\beta_{(+)} + \beta_{(-)} \right) + \sigma_s.$$

(18)

Therefore electrical resistivity, through its dependence to salinity, provides an information that is indirectly related to the hydraulic conductivity.

c. Self Potential Tomography SPT

The SP method consists in measuring passively the electrical potentials at the ground surface of the Earth or in boreholes using a couple of unpolarizable electrodes and a multi-channel DC-Voltmeter characterized by a high input impedance (>10 Mohm) and high precision (0.1 mV). The self-potential signal recorded during salt tracer experiment is the sum of two distinct contributions to the source current density. The first component is associated with the flow of ground water, which drags the excess electrical charge contained in the pore water, in the electrical diffuse layer coating the surface of the minerals [35]. This contribution is known as the streaming current density. The second component is related to the salinity gradient itself. It depends on the gradients of the activity of the charge carriers (ions) that are present in the pore water [e.g., 36,11,29]. This contribution is known as the diffusion current density. In an isotropic heterogeneous porous material, the total current density **j** (in A/m2) is the sum of a conductive current density (given by Ohm's law) plus the two components described above:

$$\mathbf{j} = -\sigma \cdot \nabla \varphi + \bar{Q}_V \mathbf{u} - \frac{k_b T}{e} \sigma \left(2t_{(+)} - 1 \right) \nabla \ln \sigma_f,$$

(19)

where φ is the electrical self-potential field (in V), σ is the electrical conductivity of the porous material (in S/m), k_b is the Boltzmann constant (1.381 x 10^{-23} J K^{-1}), T is the absolute temperature (°C), **u** represents the Darcy velocity (in m s^{-1}), $t_{(+)}$ =0.38 (for NaCl) denotes the microscopic Hittorf number of the cations in the pore water [31], and $\bar{Q}_V$ (in C m^{-3}) denotes the

excess charge density (due to the diffuse layer) of the pore water per unit pore volume. The

charge density $\overline{Q}_V$ can be accurately predicted from permeability (expressed in the m^2) according

to $\log_{10} \overline{Q}_V = -9.2 - 0.82 \log_{10} k$ [35].

The continuity equation for the electrical charge is $\nabla \cdot \mathbf{j} = 0$. Therefore the combination of

the constitutive equation and the continuity equation yields a Poisson equation for the electrical

potential [37]:

$$\nabla \cdot \sigma \nabla \varphi = \nabla \cdot \mathbf{j}_S, \tag{20}$$

$$\mathbf{j}_S = \overline{Q}_V \mathbf{u} - \frac{k_b T}{e} \sigma \left(2t_{(+)} - 1 \right) \nabla \ln \sigma_f, \tag{21}$$

with the following boundary conditions:

$$\varphi = 0 \text{ at } \Gamma_d, \ \|r\| \to \infty, \tag{22}$$

$$-\mathbf{n}.\sigma \nabla \varphi = 0 \text{ at } \Gamma_N, \tag{23}$$

the Neumann boundary condition Γ_N imposed at the insulting air-ground interface and a

Dirichlet boundary condition Γ_d at the others boundaries. The physical model described

above illustrates that the self-potential field is related to the pathways of the salt plume

migration within the aquifer and therefore to the hydraulic conductivity field.

2.2. Forward Modeling in an Heterogenous Aquifer

The salt tracer test involves injecting an amount of the salt (NaCl) into an upgradient well

and monitoring the perturbation of the electrical conductivity and self-potential associated with

the migration of the salt tracer due to the natural hydraulic gradient plus dispersion and diffusion.

The monitoring is performed until the salt tracer reaches a downstream well where the salt

concentration is sampled at eight depths. The geophysical data are performed with a combination

of electrodes located at the ground and in the wells at eight different depths (see Figure 1).

For the synthetic model, we use a heterogeneous hydraulic conductivity field between the two wells with a distance of 8 m from each other. The two wells have the same depth 15 m (Figure 1). This model is generated by the SGSIM code [38] with an anisotropic spherical variogram with a range of 10, a sill of 0.8, 75 degrees and a fraction of 0.3. The porosity is assumed constant all over the domain ($\phi = 0.35$). The values of the hydraulic properties were chosen to approximate a sandy alluvial environment [10]. The hydraulic conductivity is used to solve numerically, in staedy state conditions, the flow problem to determine the distribution of the hydraulic head and the Darcy velocity. The domain boundaries are placed 50m far enough from the study-area located between the boreholes in order to minimize their effect on the simulated hydraulic head. The hydraulic heads imposed at the boundaries on the left and right sides of the domain were selected in a manner in which the hydraulic gradient between the both boreholes is 0.02 m m^{-1}. No-flow conditions are taken for the top and bottom boundaries. The solution of the ground water flow problem is used to evaluate the advection term (see Eq. 7) and the streaming current density (see Eq.21). The advection-dispersion equation of transport for the salt was simulated by assuming a conservative injection of the saline tracer (1 kg m^{-3}) on the upgradient borehole (on the left) considered entirely screened (see Figure 1). We assume that the values of longitudinal and transverse dispersivities are 0.5 m and 0.05 m, respectively which are homogeneous and taken in concordance with the scale of the area-study (see [15] for instance). The transient transport equation is solved over a period of 100 days.

The salt transport in the aquifer has a direct impact on the distribution of the electrical conductivity of the pore water and consequently on the bulk electrical conductivity of the porous medium. A series of simulated electrical conductivity measurements has been carried out with 21 electrodes positioned on the surface and along the walls of the two boreholes (see Figure 1). The configuration technique used for the voltage measurements is the dipole-dipole array, which provide a virtual survey of the electrical conductivity perturbation during the salt plume

movement. A series of the electrical conductivity snapshots at T= [5, 10, 15 20 and 30 days] acquired to track the salt migration and to reconstruct hydraulic conductivity.

We also simulated the acquisition of self-potential measurements to record numerically the anomalies associated with the gradients of the salinity and hydraulic gradients. In real field conditions, the electrodes used previously for ERI in the downgradient well can also be used to record the electrical potential field. The self potential measurements are assumed to be collected once a day during a period of 30 days. In real field conditions, the self potential distribution has to be recorded before the injection of the electrical current used to measured the apparent electrical resistivities. More often, the electrical resistivity equipment records the self-potential data as well but this record is considered as noise by the practitioners of ERI and only used to correct the apparent electrical resistivity data. The measured self-potential data are reported with respect to a unique reference location located remotely from the study area. The SP anomalies recorded in the downgradient well provide a precious source of information in characterizing the transport parameters. Figure 2 shows the SP signature collected from the electrodes that set up in the preferential flow area present the important anomalies when the saline plume comes close to the electrodes. Because the distance between the electrical sources at the interface of the salt front and the measuring electrodes become lower. Against by areas of low hydraulic conductivity when the salinity gradient is almost negligible the SP is invariant.

3. Stochastic Joint Inversion

3.1. Description of the Algorithm

We choose a stochastic approach to predict the hydraulic conductivity fields **m** from multiple sources data **d** coming from saline concentration C, electrical resistivity imaging ERI and self-potential SP. The pilot points is used as a parameterization technique to reduce the number of the model parameters by identifying a few key-locations (called pilot points) in the

aquifer where the values of the hydraulic conductivity can be assessed to reproduce the hydrogeophysical measurements. This is done by assuming certain spatial statistical characteristic of the hydraulic conductivity field [17]. The method consists to generate from the SGSIM code [38] a hydraulic conductivity filed conditioned by the pilot points values to be perturbed at each iteration with the mean of the field. Then, the spatial distribution of the hydraulic conductivity field is written as [19]:

$$K(\mathbf{x}) = 10^{f_p(\mathbf{x})} \overline{K},$$ (25)

where $\overline{K}$ is the mean value of the hydraulic conductivity (in m s^{-1}), $f_P(\mathbf{x})$ denotes a space random function with zero as mean and the variance is equivalent as of the semi-variogram of the $Log_{10}K(\mathbf{x})$. The field is obtained by a conditional simulation from the values assigned to the pilot points; this field requires a known semi-variogram. The unknown parameters to be explored during the inverse process to reconstruct the hydraulic conductivity are $\mathbf{m} = \left[\overline{K}, f_p(\mathbf{x}) \right]$, the mean of the hydraulic conductivity $\overline{K}$ and pilot point values of $f_P(\mathbf{x})$. These parameters are perturbed at each iteration during the inverse process to honor the hydrological plus geophysical data (see Figure 3).

In a probabilistic framework, the inverse problem maximizes the conditional probability density of occurring $\mathbf{m}$ given the data vector $\mathbf{d}$. This corresponds to find the best model $\mathbf{m}$ respecting the constraints posed by the prior model and minimizing the difference between the observed and predicted data. We note $P_0(\mathbf{m})$ is the prior probability density of parameters $\mathbf{m}$ and $P(\mathbf{d}|\mathbf{m})$ represents the probability corresponding to the data for $\mathbf{m}$ fixed, as it permits to establish a connection between the predicted data from the model $\mathbf{m}$ and measurable data $\mathbf{d}$. The posteriori probability density $\pi(\mathbf{m}|\mathbf{d})$ of the model parameters $\mathbf{m}$ given the data $\mathbf{d}$ is obtained using Bayes formula:

$$\pi(\mathbf{m}|\mathbf{d}) = \frac{P(\mathbf{d}|\mathbf{m})P_0(\mathbf{m})}{P(\mathbf{d})}, \tag{26}$$

where $P(\mathbf{d})$ is a normalizing term known as evidence,

$$P(\mathbf{d}|M) = \int P_0(\mathbf{m}|M)P(\mathbf{d}|\mathbf{m}, M)d\mathbf{m}. \tag{27}$$

The posteriori probability density $\pi(\mathbf{m}|\mathbf{d})$ of the model parameters $\mathbf{m}$ given the data $\mathbf{d}$ is written as:

$$\pi(\mathbf{m}|\mathbf{d}) \propto P(\mathbf{d}|\mathbf{m})P_0(\mathbf{m}). \tag{28}$$

The presence of the multiple and independents data sets require rewriting the likelihood function as a product of the partial likelihood of each data type:

$$P(\mathbf{d}_i|\mathbf{m}) = \prod_i^n P_i(\mathbf{d}_i|\mathbf{m}), \tag{29}$$

$$\pi(\mathbf{m}|\mathbf{d}_i) \propto \left[\prod_i^n P_i(\mathbf{d}_i|\mathbf{m}) \right] P_0(\mathbf{m}), \tag{30}$$

where i numerated the data set in this case, we have three types of data (salt concentration, C, ERI and SP). The likelihood function used to evaluate the quality of a model $\mathbf{m}$ to maximize a probability function considered to be Gaussian:

$$P_i(\mathbf{d}_i/\mathbf{m}) = \frac{1}{[(2\pi)^N \det C_{d_i}]^{1/2}} \exp\left[-\frac{1}{2}(g_i(\mathbf{m})-\mathbf{d}_i)^T C_{d_i}^{-1}(g_i(\mathbf{m})-\mathbf{d}_i) \right], \tag{31}$$

where $g_i(\mathbf{m})$ is the forward modeling operator to simulate a type of data referenced by i. The forward problem connects the generation of data $\mathbf{d}$ to a given hydraulic conductivity model $\mathbf{m}$, $\mathbf{d}$ is a N-vector corresponding to the observed data, and C_{d_i} denotes a ($N \times N$) diagonal covariance matrix to account for the noise-to-signal ratio of the data in the inversion. In the salt tracer test, the forward problem is composed of three partial forward problems, the first forward problem $g_1(\mathbf{m})$ allowed to generate saline concentration, the second $g_2(\mathbf{m})$ to simulate the electrical

resistivity imaging and the third $g_3(\mathbf{m})$ for the self potential signals. The prior probability distribution, if available, is also taken Gaussian:

$$P_0(\mathbf{m}) = \frac{1}{[(2\pi)^M \det C_m]^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{m} - \mathbf{m}_{prior})^T C_m^{-1}(\mathbf{m} - \mathbf{m}_{prior})\right], \qquad (32)$$

where $\mathbf{m}_{prior}$ is the prior value of the model parameters for each unit and C_m is the model diagonal covariance matrix incorporating the uncertainties related of the prior model of material properties.

To avoid the unrealistic values of the hydraulic conductivity, the inequality constraints $\mathbf{b}_{\min} \leq S(\mathbf{m}) \leq \mathbf{b}_{\max}$ were included in the posteriori probability. Therefore, we rewrite the posteriori probability distribution constrained as following [39]:

$$\pi(\mathbf{m}|\mathbf{d}) \propto P(\mathbf{d}|\mathbf{m})P_0(\mathbf{m})\left[\mathcal{H}(S(\mathbf{m}) - \mathbf{b}_{\min})^T \mathcal{H}(\mathbf{b}_{\max} - S(\mathbf{m}))\right], \qquad (33)$$

where,

$$\mathcal{H}(S(\mathbf{m}) - \mathbf{b}_{\min}) = \begin{cases} 1 & S(\mathbf{m}) > \mathbf{b}_{\min} \\ 1/2 & S(\mathbf{m}) = \mathbf{b}_{\min} \\ 0 & S(\mathbf{m}) < \mathbf{b}_{\min} \end{cases}, \qquad (34)$$

where $\mathcal{H}$ denotes the Heaviside function, $K(x) = S(\mathbf{m})$ is function to generate the hydraulic conductivity field with the parameters $\mathbf{m}$ ($\overline{K}$ and $f_P(\mathbf{x})$). $\mathbf{b}_{\max}$ and $\mathbf{b}_{\min}$ are the lower and upper limits of the hydraulic conductivity values. We point out that the posterior probability constrained (Eq. 33) is going to be non-Gaussian.

In this paper, the pilot points are placed on a regular grid, where their unknown values can be assess with the mean of the hydraulic conductivity $\overline{K}$ through a Markov chain Monte Carlo (McMC) algorithm via the exploration of the posteriori probability density $\pi(\mathbf{m}|\mathbf{d})$, which is expressed by Eq. (33). McMC algorithms are based on the random generation routines of a set of the models (in the present case the values of the pilot point and the mean of the hydraulic conductivity $\overline{K}$) (for more details about the McMC algorithm with bound constraints see the

appendix A). Then we select the best realizations of these models able to reproduce the hyrogeological plus geophysical data (see Figure 4). We can infer the statistic properties of the ensemble of the realizations such as the central tendency (median, mode. and mean) and the percentiles as dispersion parameter to estimate the uncertainty.

3.2. Results

To evaluate the effectiveness of the inversion with coupling hydrogeological and geophysical data, we compared several scenarios. We first examined the individual inversion of each type of data, and then we proceed to combine two data sets. At the end, we combined all the hydrogeological and geophysical datasets altogether (time-lapse joint inversion).

Note that the inversion was performed with an inequality constraint on the hydraulic conductivity field with 10^{-7} and $10^{-1.5}$ m s^{-1} as the upper and lower limit respectively to facilitate a reasonable sampling of this field with a set of the 49 pilot points distributed on the regular grid (Figure 3). The results of the inversion are presented from the computation of the median and the both percentiles 25% and 75% of the simulations sampled (for example see Figure 5). The discussion of the inversion is based on a comparison between the real and inverted hydraulic conductivity, which is calculated from the median of the simulations sampled via posterior probability. We compared the spatial distribution of the real salted plume and the plume resulting from the hydraulic conductivity inverted at time $t = 20$ days, we also added two saline fronts are determined respectively from the fields of hydraulic conductivity of 75% and 25% quartiles of simulations sampled. As, the discussion included the predicted tracer concentration curve collected at z=6m in the downgradient borehole and the predicted self potential curve obtained from the electrodes of the downgradient borehole after 20 days the saline injection. In the inversion procedure, the variogram parameters and porosity are assumed known.

At first, we evaluated the reconstruction of the hydraulic conductivity field from the salt concentration data collected at the downgradient borehole at eight depths (Figure 3) during for

the time lapse comprised between *t=0* to *t=100* days. The 20,000 simulations of the hydraulic conductivity fields were generated during the McMC process. The median field of the realizations sampled from a posteriori distribution varies between 10^{-7} and $10^{-1.8}$ m s^{-1} [the results of this inversion called *inversion A1,* see Figure 6a]. The median of the realizations sampled honors perfectly the saline concentration data (see Figure 7a). However, the spatial distribution of the salty plume at *t=20* days reconstructed from this median field of the hydraulic conductivity and two quartiles are far to be close to the true position of the saline plume (Figure 6b). We found that the SP signature predicted from this field remains near to the SP observed (Figure 7b). We note that the inversion of the concentration data did not delineate the main permeable areas. That said, there is a lack of the similarity between the inverted hydraulic conductivity field and the true one (Figure 6c). Consequently, the downstream salt concentration measurements in a well do not bring enough information to reconstruct the hydraulic conductivity field.

Regarding the electrical resistivity method, we used a series of electrodes settled in the two wells with a Dipole-Dipole array (Figure 3). The data has been acquired during the following periods (in days) [5, 10, 15, 20 and 30] to follow the variations of the electrical resistivity associated with the passage of the salt tracer. [The results of this inversion called *inversion A2,* see figure 6]; the hydraulic conductivity estimated by using the inversion of the resistivity data are far from perfect according to difference between the true and inverted fields (figure 6c). In the top surface of the aquifer, where the hydraulic conductivity takes high values, the reconstructed hydraulic conductivity field generates an erroneous distribution of the saline tracer (see Figure 7c).

The inversion of the self-potential measurements, recorded once per day during a period of 30 days on the same places of the ERI electrodes, yields a hydraulic conductivity field in the range [10^{-2} m s^{-1}, 10^{-7} m s^{-1}] [called *inversion A2,* see figure 6]. Visually, this reconstruction also reproduces the concentration data (Figure 6e). This reconstruction is better than the reconstructed

field using the DC resistivity data. The hydraulic conductivity field inverted from the SP data yield an area of lower hydraulic conductivity in the vicinity of the upgradient well between the depths [-2 m, -6 m] that does not exist in the true conductivity field (Figure 6f). That said, the result is still far from the true field according to the difference between the true and inverted fields (figure 6f).

Now, we present the inversion results with the data taken two by two (Figure 7). We emphasize that the result of the joint inversion of apparent resistivity and concentration data (*inversion B1*) improves considerably the mapping of the hydraulic conductivity compared to results obtained from the decoupled inversion data of the concentration and ERI (see *inversion B1* Figure 8 and compare with Figure 6 *inversions A1& A2*). In this joint inversion, we eliminated the occurrence of the area of high conductivity hydraulic near to surface obtained with the inversion decoupled of the resistivity apparent. However, the spatial distribution of salt at (*t=20* days) reveals that the estimated hydraulic conductivity does not reproduce perfectly the salt motion in the porous medium (Figure 8b).

The joint inversion of concentration data and self potential (*inversion B2*) delineates pretty well the area of preferential flows located in the center of the domain (see Figure 8d). This is also seen on the results of the joint inversion of the ERI and SP data (see *inversion B3* Figure 8g): the spatial distribution of hydraulic conductivity and the trajectory of the salt plume have been improved by the joint inversion (Figure 8g). We emphasize that the predicted data on the concentration and the self potential were bests fitted than the uncoupled inversions (Figure 9).

The joint inversion of the self-potential, resistivity, and salinities altogether (*inversion C*) have improved the reconstruction of the hydraulic conductivity field due to the dual sensitivity of the tool to salinity and pressure hydraulic gradients. Regarding, the simultaneous inversion of all data as shown in the Figure 10, then, without doubt it's the best strategy to increase the resolution for best coverage of the all domain studied and constrained the optimization to converge to the

reliable model. Usually, the hydraulic imaging can be improved from the combining of the two or more sets of the data.

4. Concluding Statements

We have presented a synthetic case study aiming at see the usefulness of adding self potential measurements in addition to electrical resistivity imaging and in situ concentration measurements to reconstruct the hydraulic conductivity field of an heterogenous aquifer during a salt tracer test. As most of the impedance meters used for ERI are also able to record the self-potential field before the injection of electric current, this approach does not require new equipments with respect to those used traditionally in hydrogeophysics. Despite the fact that self-potential is considered as a source of noise in ERI studies, we show that it contains valuable information because of the influence of the salt tracer upon the source current density responsible for the occurrence of the self-potential signals. In our study, we showed that self-potential could be a complementary tool to investigate salt tracer tests and to reconstruct the hydraulic conductivity field.

In our synthetic experiment, the salt tracer was tracked directly by the sampling of the salinity concentration in the down-gradient well. The tracking of the salt was carried out indirectly thanks to a combination of ERI and SP measurements. The movement of the salt plume perturbs linearly the background electrical conductivity. Consequently the ERI imaging permits to draw the limit of the salt front at different times. The hydraulic and salinity gradients in the ground are responsible for the creation of the self potential signature with a magnitude that is easily measurable at the ground surface and in boreholes. Finally, we have applied a stochastic approach to jointly invert different data sets to reconstruct the hydraulic conductivity field. To reduce the numbers of the unknown parameters to map the hydraulic conductivity and the time of computation, we have used the pilot point method as a technique of parameterization. To

improve the efficiency of this approach; we choose to introduce constraints on the hydraulic conductivity values to avoid the sampling of unrealistic values of the hydraulic conductivity. A future work will concern the inclusion of the spectral induced polarization method (because of its sensitivity to permeability, see Revil and Florsch, [40] and references therein) and to reduce the computation time we will parallelize the computation.

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Appendix. A:

A description of the Metropolis-Hastings algorithm with inequality constraint:

Step 1: Specify an arbitrary starting value $\mathbf{m}^0$ which satisfies the constraints. Set $i = 0$.

Step 2: Given the current value $\mathbf{m}^i$, use a symmetric transition density $q(\mathbf{m}^i|\mathbf{m}^c)$ to generate a candidate for the next value in the sequence, $\mathbf{m}^c$.

Step 3: Use the candidate value $\mathbf{m}^c$ to evaluate the inequality constraints $\left[\mathcal{H}(S(\mathbf{m}) - \mathbf{b}_{min})^T \mathcal{H}(\mathbf{b}_{max} - S(\mathbf{m})) \right]$. If any constraints are violated set $\alpha(\mathbf{m}^i; \mathbf{m}^c) = 0$ and go to **Step 5**.

Step 4: Calculate $\alpha(\mathbf{m}^i; \mathbf{m}^c) = \min\left[1, \dfrac{\pi(\mathbf{m}^c)q(\mathbf{m}^c|\mathbf{m}^i)}{\pi(\mathbf{m}^i)q(\mathbf{m}^i|\mathbf{m}^c)} \right]$

Step 5: Generate an independent uniform random variable U from the interval [0, 1].

Step 6: Set $\mathbf{m}^{i+1} = \begin{cases} \mathbf{m}^c, & \text{if } U < \alpha(\mathbf{m}^i, \mathbf{m}^c) \\ \mathbf{m}^i, & \text{if } U \geq \alpha(\mathbf{m}^i, \mathbf{m}^c) \end{cases}$

Step 7: Set $i = i+1$ and go to **Step 2**.
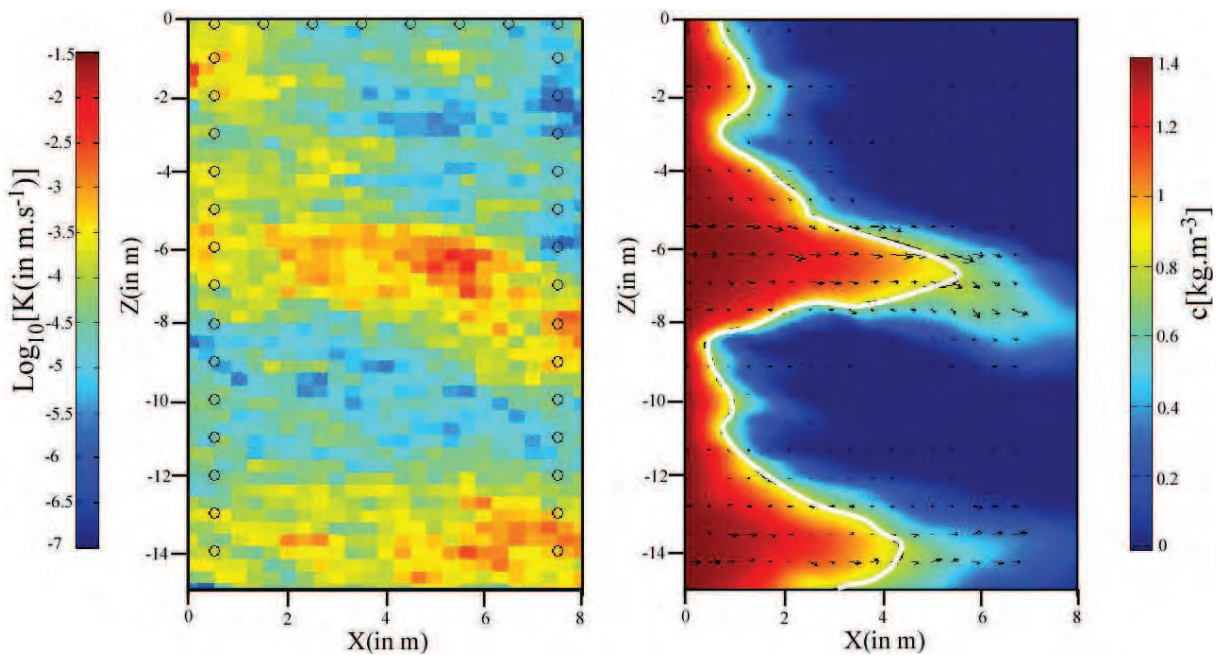
Figure 1: The "true" hydraulic conductivity field generated by SGSIM code to assess the efficiency of the stochastic joint inversion to build this field through the joint inversion of the three data sets (self potential, the apparent resistivity and the saline concentration data) are recorded on the both boreholes in the time lapse mode to track the salt movement. The salt movement is due to natural hydraulic gradient between the upgradient borehole (on the left) and the downgradient borehole (on the right). The salt distribution at $t = 20$ days showing the preferential area the salt transport corresponding to the high hydraulic conductivity area.
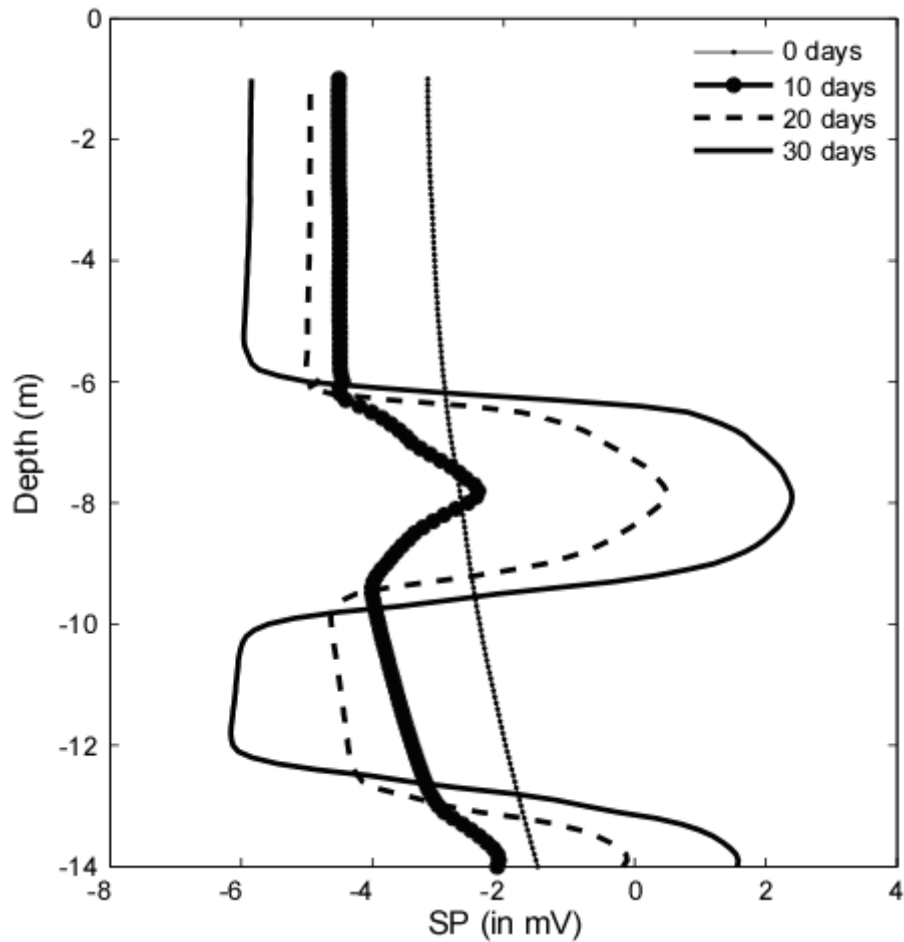
Figure 2: The self-potential signatures collected during the fourth periods at the electrodes located in downgradient borehole. The curves illustrate that the SP anomalies present the important anomalies when the feather comes close to the electrodes. Because the distance between the electrical sources at the interface of the salt front and the measuring electrodes become lower. The SP remains invariant in the area characterized by the lower hydraulic conductivity values.
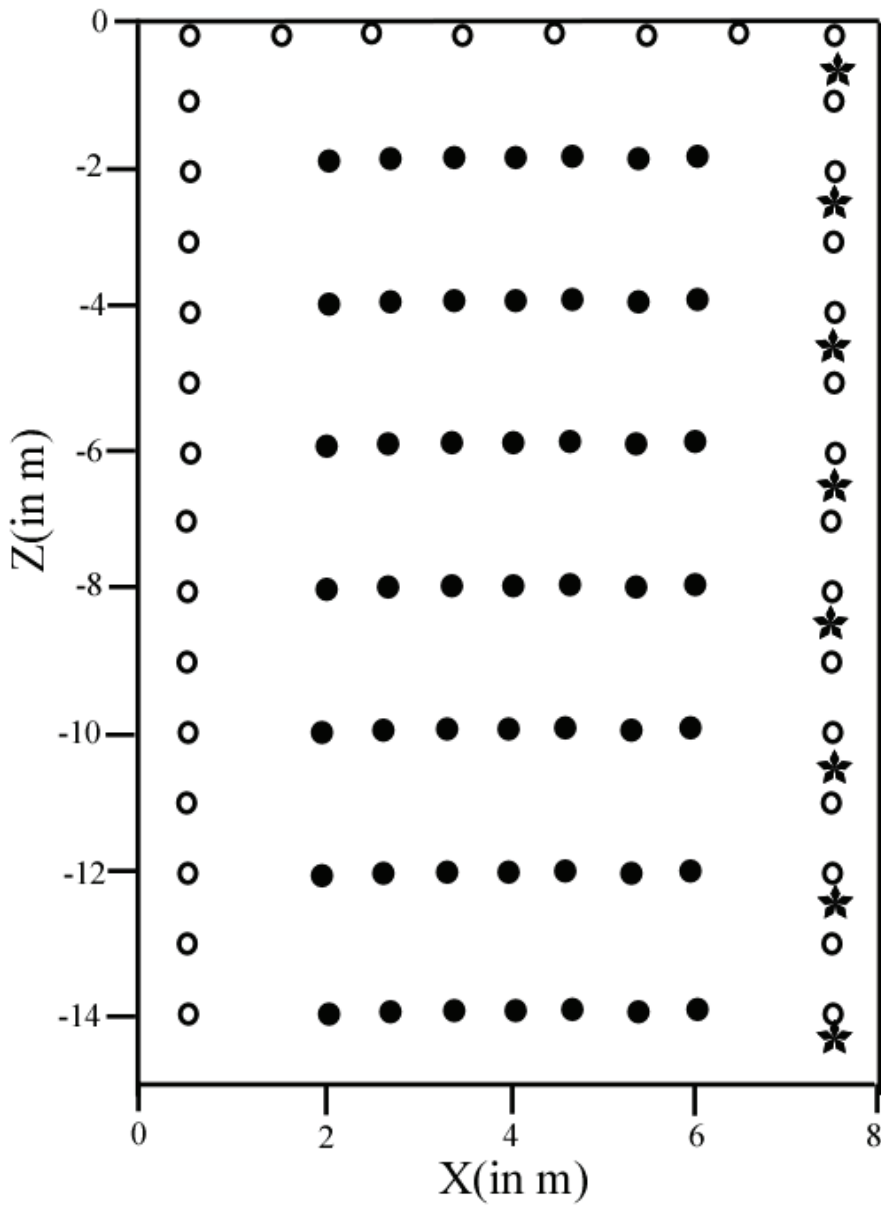
Figure 3. The filled circles indicate the spatial locations of the pilot points where we assign the hydraulic conductivity values selected randomly during each iteration of the McMC sampler to build the field perturbation by using conditional simulation with a variogram assumed to be known. The white circles are the locations of electrodes in both boreholes to monitor the variations in time of the electrical resistivity and self potential. The stars indicate the position of monitoring of the changes of salt concentration.
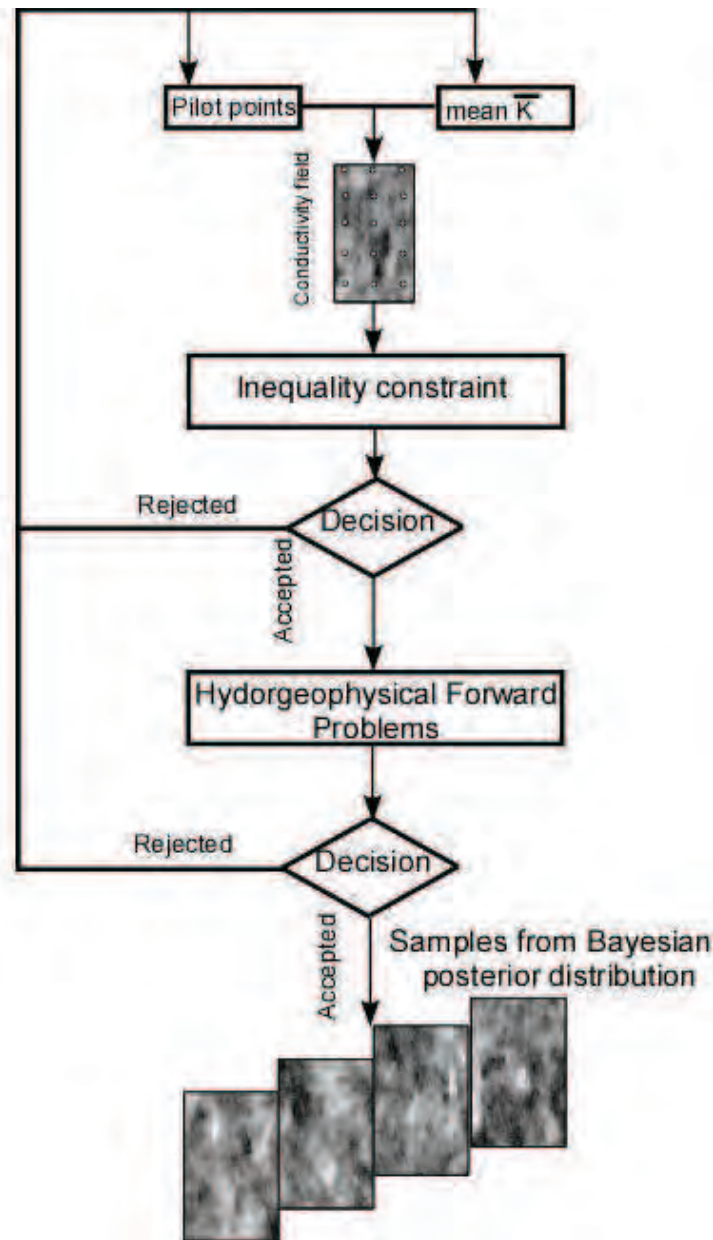
Figure 4. The graphical illustration of the McMC algorithm which was used for reconstruct the hydraulic conductivity from the inversion of the hydrogeophysical data (tracer concentration, resistivity apparent data and self-potential). The pilot points technique used as parameterization method to perturb the mean hydraulic conductivity $\overline{K}$ supposed also as unknown parameter. The prior model is introduced as inequality constraints to avoid the sampling of undesirable hydraulic conductivity values. The protocol provides a set of the realizations sampled from the posterior probability distribution.
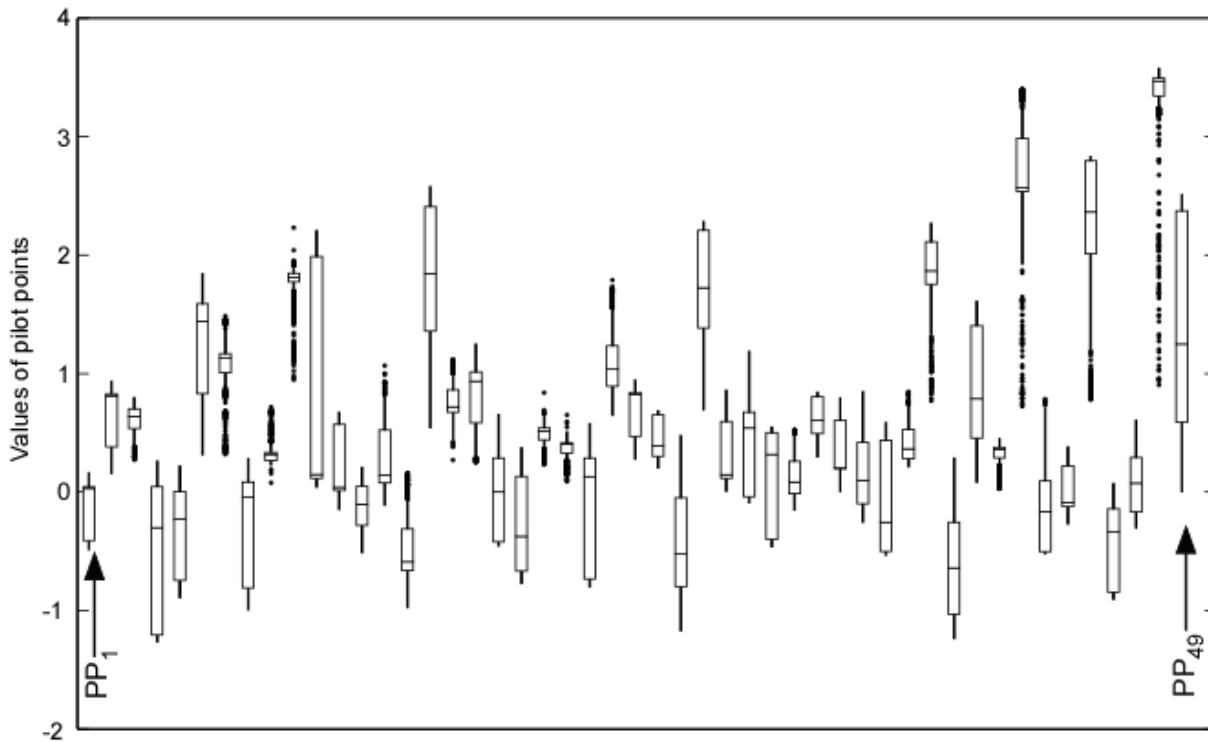
Figure 5. The box plots showing the results of the 20,000 realizations of the values of the each pilot point (from 1 to 49 pilot points) obtained from the McMC sampling to invert the concentration saline data. The box plots represent inter-quartile ranges and the solid black line at the centre of each box is the median; the arms of each box extend to cover the central 95 per cent of the distribution.

Log$_{10}$ K inverted   salt concentration at T=20 days   Difference (Ktrue-Kinverted)
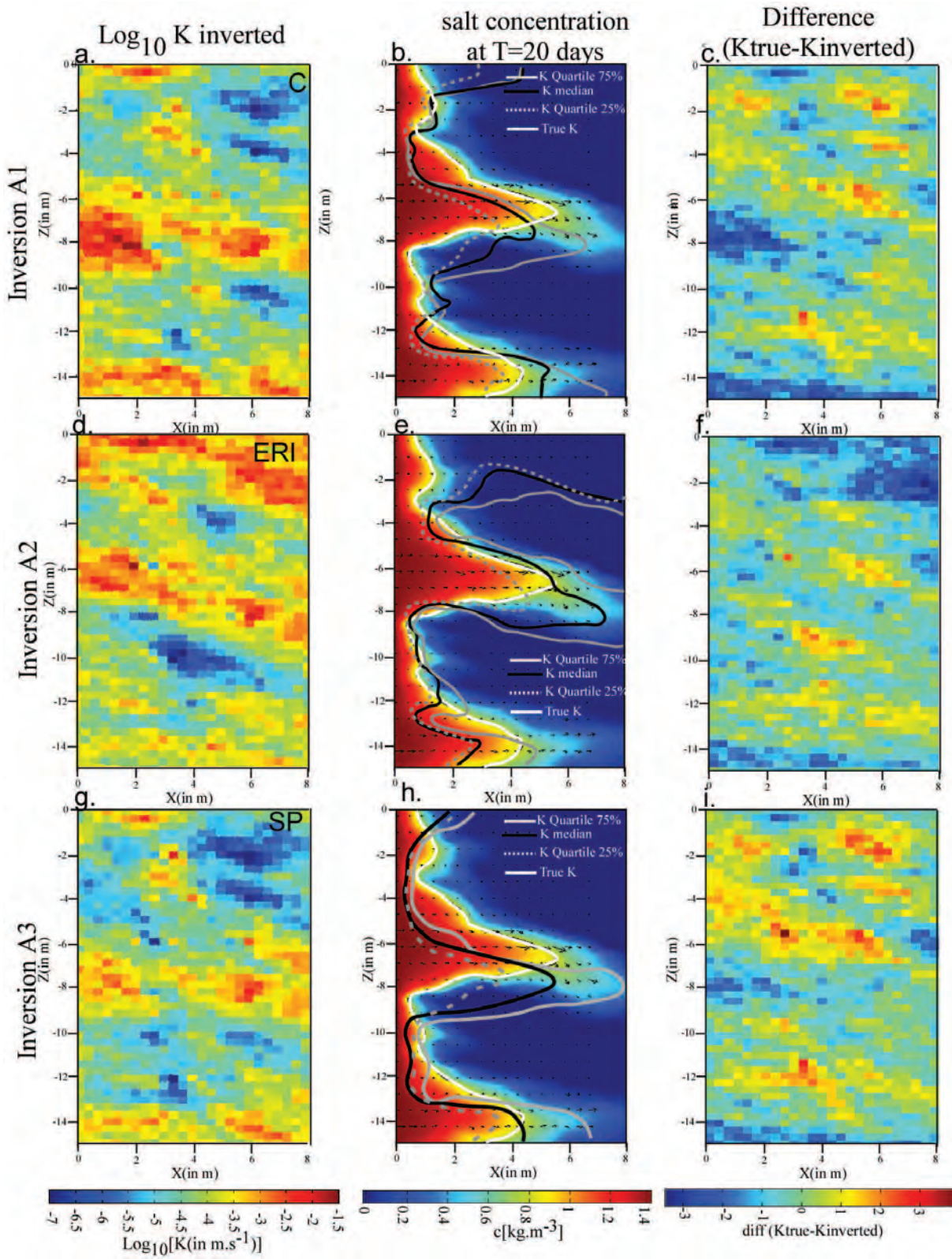
Figure 6. The results of the uncoupled inversion of each data type to estimate the hydraulic conductivity field. The inversion A1, A2 and A3 designate the inversion of concentration data, resistivity and self potential data respectively. For each inversion we represent: in the first column the hydraulic conductivity inverted (Figures a, d and g), in the second column (Figures b, e and h) the distribution of the salt plume at $t=20$ days calculated with the real hydraulic conductivity field, we added the results of the spatial distribution of the salt front calculated from the median and the quartiles 75% and 25% of all realizations sampled during the inversions. In the third column (Figures c, f and i), the difference between the hydraulic conductivity of the real field and the inverted filed.
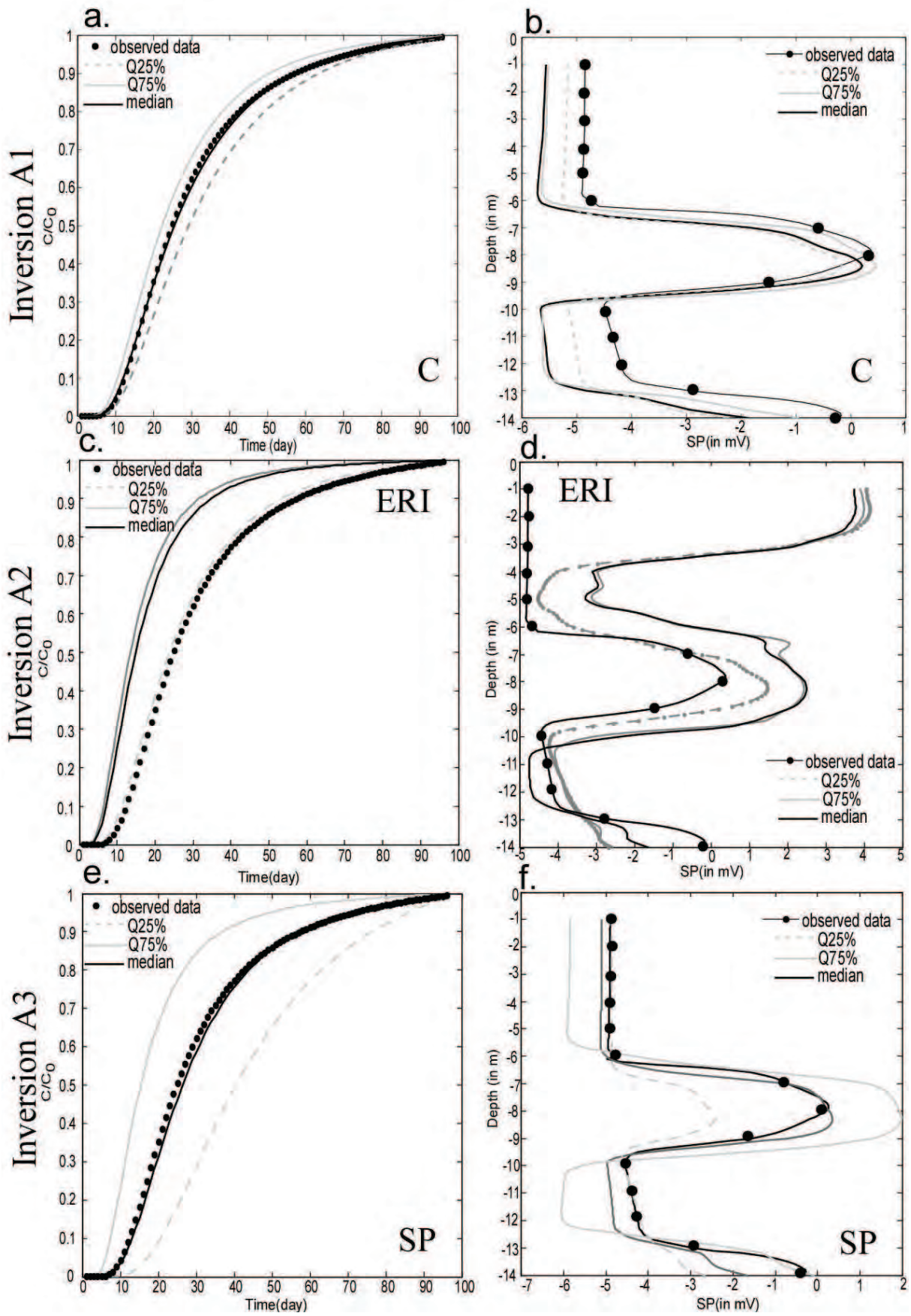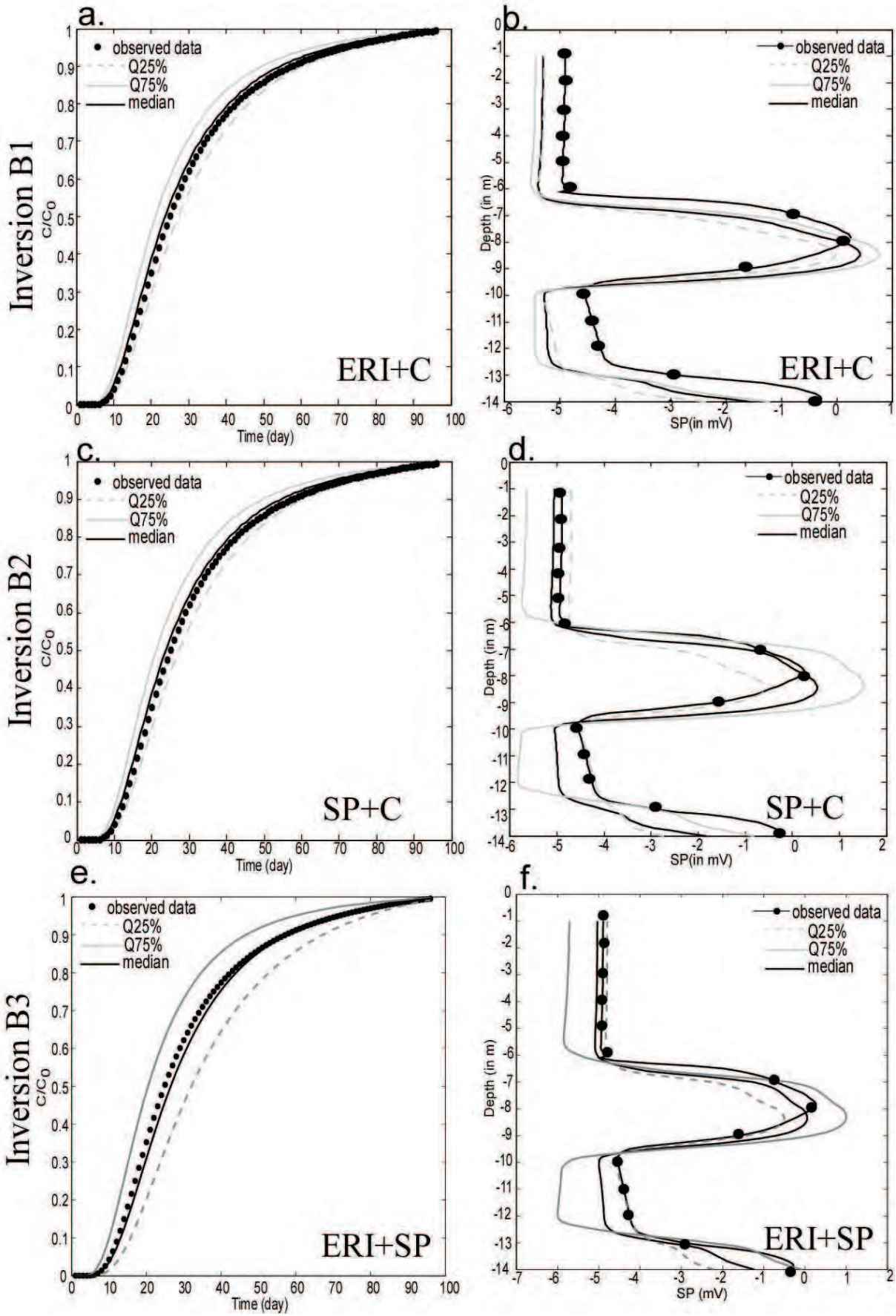
Figure 7. The inversion A1, A2 and A3 indicate the inversion of concentration data, resistivity and self potential data respectively. For each inversion, we illustrate in the first and second columns the predicated data of the saline concentration at position [x=-7.5, z=-8 m] and SP data collected at the downgradient borehole determinate from the inverted hydraulic conductivity field.
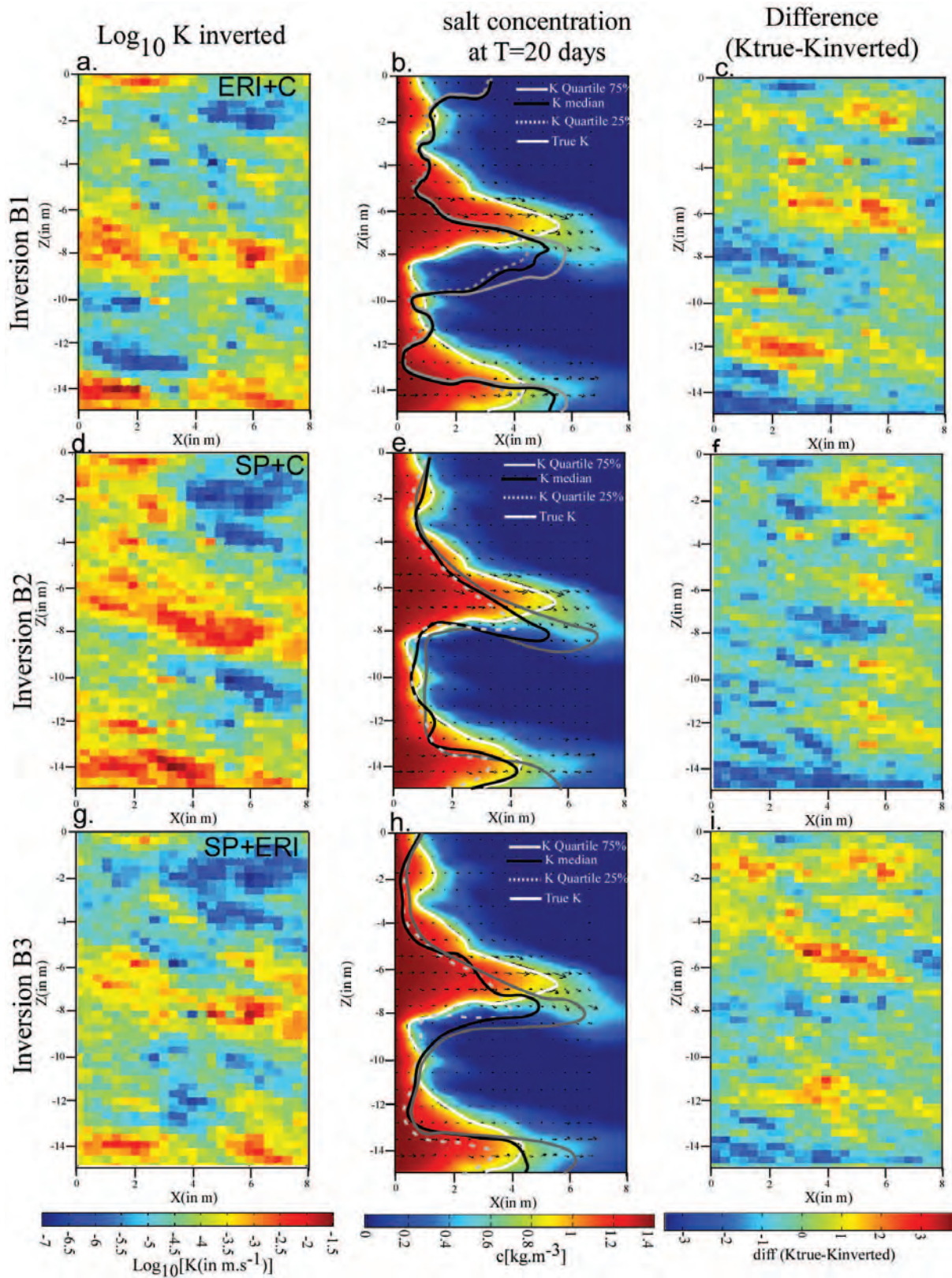
Figure 8. The results of the coupled inversion of both data sets to estimate the hydraulic conductivity field. The inversion B1, B2 and B3 reference the inversion of the ERI+C, SP+C and ERI+SP respectively. The figures in the first column (Figures a, d and g) represent respectively

the hydraulic conductivity fields determined from the coupled inversion. The figures in the second column (Figures b, e and h), on the distribution of the salt plume at *t=20* days calculated with the real hydraulic conductivity field, we added the results of the spatial distribution of the salt front calculated from the median and the quartiles 75% and 25% of all realizations sampled during the inversion. The third column for each inversion we represent the difference between the true hydraulic conductivity field and the hydraulic conductivity inverted.

Figure 9. The inversion B1, B2 and B3 indicate the coupled inversions (ERI+C, SP+C and ERI +SP respectively). For each inversion, we represent in the first column the predicted data of the concentration sampled at position [x=-7.5, z=-8 m] and in the second column the predicted data of SP data collected at the downgradient borehole at *t=20* days.
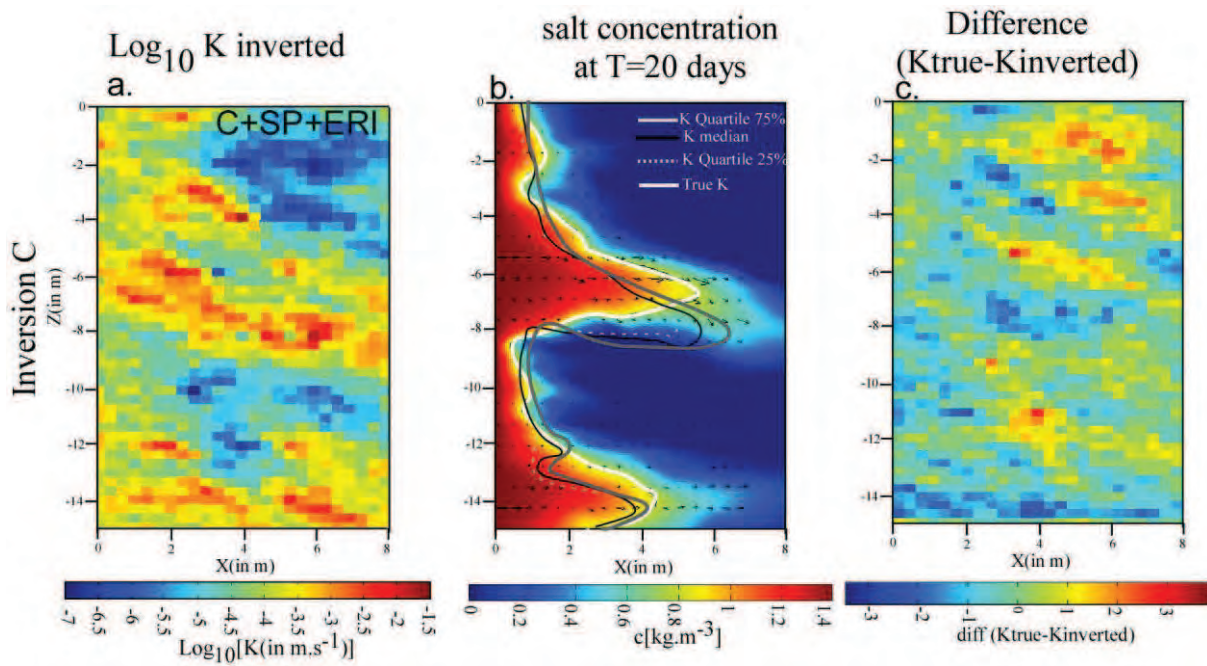
Figure 10. The hydraulic conductivity filed estimated from the joint inversion of all data sets (C, ERI and SP). **b.** the spatial distribution of the saline plume at 20 days of the injection from the true conductivity filed compared to plume resulting from the inversion with the confidence interval. **c.** the difference between the true hydraulic conductivity distribution and the inverted filed.
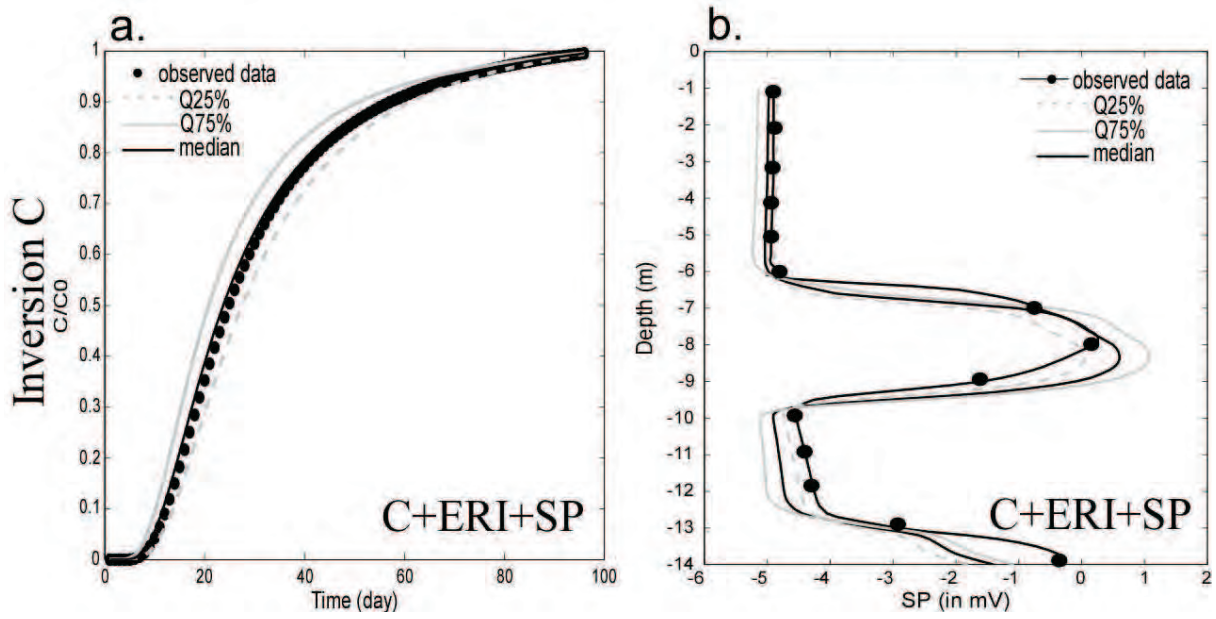
Figure 11. The predicted data obtained from the joint inversion of all data sets. **a** and **b** correspond to the predicted data of the saline concentration recorded at [x=7.5, z=-8] and SP data recorded at the down gradient boreholes at T=20 days.

Annexe 3:

The forward problem of streaming potential in the unsaturated conditions involves the combining of the Richards equation and Poisson equation in the quasi-static conditions. We consider a geometrical domain Ω corresponding to the subsurface domain under the influence of the tide (see Figure 20). The tidal forcing is imposed using a Dirichlet boundary condition at the right border Γ (where the estuary of the river is located) to solve Richards equation. Richards equation is given by,

$$\left[\frac{C}{\rho_f g} + S_e S_s\right]\frac{\partial p}{\partial t} + \nabla \cdot \mathbf{u} = 0, \tag{1}$$

where the Darcy velocity $\mathbf{u}$ is given by,

$$\mathbf{u} = -\frac{k_s}{\mu}k_r\nabla(p + \rho_f gz), \tag{2}$$

$$H_p = p / \rho_f g, \tag{3}$$

The Richards's equation is solved subject to the following boundary conditions:

$$\mathbf{n} \cdot \left[\frac{k_s}{\mu}k_r\nabla(p + \rho_f gz)\right] = 0, \tag{4}$$

$$p = \rho_f gh_t, \tag{5}$$

where the pore fluid pressure p (in pa) is the variable to solve, C_e denotes the specific moisture capacity (in m^{-1}) defined by $C_e = \partial\theta / \partial H_p$, θ denotes the water content (dimensionless), S_e denotes the effective saturation (dimensionless) $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$, θ_r is the residual water content, θ_s denotes the porosity, H_p denotes the pressure head (m), k_r the relative permeability, k_s the permeability at saturation, $\mathbf{n}$ the unit vector normal to the boundary, and h_t denotes the tidal head (in m) of the Seine River imposed as Dirichlet boundary as discussed above.

With the van Genuchten parameterization, we consider the soil to be saturated when the fluid pressure reaches the atmospheric pressure ($H_p = 0$). The effective saturation, the specific moisture capacity, and the relative permeability are given by,

$$S_e = \begin{cases} 1/\left[1+\left|\alpha H_p\right|^n\right]^m, & H_p < 0 \\ 1, & H_p \geq 0 \end{cases}, \tag{6}$$

$$C_e = \begin{cases} \dfrac{\alpha m}{1-m}\ \varphi - \theta_r\ S_e^{\frac{1}{m}}\left(1-S_e^{\frac{1}{m}}\right)^m, & H_p < 0 \\ 0, & H_p \geq 0 \end{cases}, \tag{7}$$

$$k_r = \begin{cases} S_e^L\left[1-\left(1-S_e^{\frac{1}{m}}\right)^m\right]^2, & H_p < 0 \\ 1, & H_p \geq 0 \end{cases}, \tag{8}$$

where α, n, $m = 1 - 1/n$, and L are dimensionless constants that characterize the porous material [*van Genuchten*, 1980].

In an isotropic heterogeneous media, the total current density $\mathbf{j}$ (in A/m2) is the sum of a conductive current density (given by Ohm's law) plus electrokinetic source (e.g., Linde et al., [2007]):

$$\mathbf{j} = -\overline{\sigma}(S_e) \cdot \nabla\varphi + \frac{\overline{Q}_V}{S_e}\mathbf{u}, \tag{9}$$

where φ is the electrical self-potential (in V), σ is the electrical conductivity of the porous material (in S/m), which depends to the water content, $\overline{Q}_V$ (in C m^{-3}) is the excess charge density (due to the diffuse layer) of the pore water per unit pore volume. The charge density $\overline{Q}_V$ can be predicted from permeability (in m^2) according to $\log_{10}\overline{Q}_V = -9.2 - 0.82\log_{10}k_s$ (*Jardani and Revil.*, [2009]).

The continuity equation for the electrical charge is $\nabla \cdot \mathbf{j} = 0$ and the electrical (self-) potential is therefore obtained by solving the following Poisson equation

$$\nabla \cdot \sigma(S_e)\nabla\varphi = \nabla \cdot \mathbf{j}_S,\tag{10}$$

$$\mathbf{j}_S = \frac{\overline{Q}_V}{S_e}\mathbf{u},\tag{11}$$

with the following boundary conditions:

$$\varphi = 0 \ \text{at} \ \Gamma_d, \ \|r\| \to \infty,\tag{12}$$

$$-\boldsymbol{n}.\sigma\nabla\varphi = 0 \ \text{at} \ \Gamma_N,\tag{13}$$

The physical model described illustrates that the signature of the self potential is strongly linked to the pathways of the flux water (*Revil and Jardani*., [2010]).

Stochastic inversion of permeability and dispersivities from time lapse self-potential measurements: A controlled sandbox study

A. Revil[1,2] and A. Jardani[3]

[1] We test the ability of the self-potential method to provide information that can be used to invert the permeability and the dispersivities of a porous material. We first formulate the semi-coupled system of equations describing the occurrence of self-potential signals associated with the transport of salt in a porous material like sand. Two contributions in the source current density arise from the electrokinetic coupling associated with the flow of the pore water and the diffusion potential due to the gradient in the activity of the salt. A controlled laboratory sandbox experiment is performed to show that time-lapse self-potential measurements can be used to invert, in a Bayesian framework, the permeability and dispersivities. **Citation:** Revil, A., and A. Jardani (2010), Stochastic inversion of permeability and dispersivities from time lapse self-potential measurements: A controlled sandbox study, *Geophys. Res. Lett.*, *37*, L11404, doi:10.1029/2010GL043257.

1. Introduction

[2] Imaging aquifer transport properties is an important task in hydrogeophysics. The non-intrusive monitoring of salt plume is traditionally done using GPR or electrical resistivity tomography with the goal of inferring the distribution of the permeability and dispersivities. In this letter, we are looking for a passive (real-time) monitoring and non-intrusive geophysical method to monitor a salt tracer test. The expression "non-intrusive" is used below to say that we measure remotely the geophysical signal generated by the presence of a plume of salt water. In our approach, the presence of a salt plume generates indeed an electrical current on its own and this electrical current generates in turn an electrical field called the self-potential field. *Maineult et al.* [2005] showed that the migration of a salt plume creates an electrical field that can be explained by combining electro-kinetic and diffusion potential theories. However they put their electrodes directly in contact with the salt plume, so their measurements were intrusive. While this was a good start, we feel that the self-potential method can be used remotely to investigate the plume properties over time.

2. Background Theory

[3] The constitutive equations are Darcy's law for the Darcy velocity **u** (in m s^{-1}), Fick's law for the flux of the salt $\mathbf{j}_d$ (in kg m^{-2} s^{-1}), and Ohm's law for the current density **j** (in A m^{-2}),

$$\mathbf{u} = \phi\mathbf{v} = -\frac{1}{\eta_f}\mathbf{k} \cdot \left(\nabla p + \rho_f g\nabla z\right), \tag{1}$$

$$\mathbf{j}_d = -\rho_f\phi\mathbf{D} \cdot \nabla C_m + \rho_f\phi\mathbf{v}C_m, \tag{2}$$

$$\mathbf{j} = -\sigma \cdot \nabla\varphi + \bar{Q}_V\mathbf{u} - k_b T\sum_{i=1}^{N}\frac{T_i}{q_i}\sigma \cdot \nabla\ln\{i\}, \tag{3}$$

(see *Oltean and Buès* [2002] for equations (1) and (2) and *Revil and Linde* [2006] for equation (3)) where **v** is the mean velocity of the pore water (m s^{-1}), **k** (in m^2) is the permeability tensor, **D** (in m^{2}s^{-1}) is the hydrodynamic dispersion tensor, $\sigma = \sigma\,\mathbf{I}_3$ is the electrical conductivity tensor (in S m^{-1}), $\mathbf{I}_3$ is the 3 × 3 unit tensor, ϕ [−] is the connected porosity, p is the pore pressure (Pa), φ is the self-potential (in V), C_m is the solute mass fraction (dimensionless), η_f is the dynamic viscosity of the fluid (in Pa s), ρ_f is the solute bulk density (in kg m^{-3}), $g = 9.81$ m s^{-2}, k_b is the Boltzmann constant (1.381 × 10^{-23} J K^{-1}), q_i is the charge (in C) of species i dissolved in water $\{i\}$ is the activity of species i, T_i (dimensionless) is the macroscopic Hittorf number of the ionic species i in the porous material (that is the fraction of electrical current carried by species i), and $\{i\}$ (in m^{-3}) represents the activity of the ionic species i, and $\bar{Q}_V$ (in C m^{-3}) is the excess charge density (due to the diffuse layer) of the pore water per unit pore volume. The charge density $\bar{Q}_V$ can be predicted from permeability according to $\log_{10}\bar{Q}_V = -9.2 - 0.82\log_{10}k$ [*Jardani and Revil*, 2009].

[4] In addition to the constitutive equations, we have to consider three continuity equations for the mass of the pore water, for the mass of the salt, and for the electrical charge,

$$\nabla \cdot (\rho_f\mathbf{u}) = -\frac{\partial(\rho_f\phi)}{\partial t} + \rho_f Q_s \tag{4}$$

$$\nabla \cdot \mathbf{j}_d = -\frac{\partial\left(\rho_f\phi C_m\right)}{\partial t} + \rho_f Q_s C_m^0, \tag{5}$$

$$\nabla \cdot \mathbf{j} = 0, \tag{6}$$

where C_m^0 is the solute mass fraction of the salt in the source term, Q_S is a volumetric hydraulic source term for the injection/abstraction of water (in s^{-1}). The effect of the salt concentration on the mass density and viscosity are given by *Oltean and Buès* [2002]: $\rho_f = \rho_f^0 + \gamma C_m$ and $\eta_f = \eta_f^0(1 + \tau_1 C_m + \tau_2 C_m^2 + \tau_3 C_m^3)$, where ρ_f^0 and η_f^0 are the mass density and viscosity of pure water, respectively, γ, τ_1, τ_2, τ_3 are empirical constants.

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a.
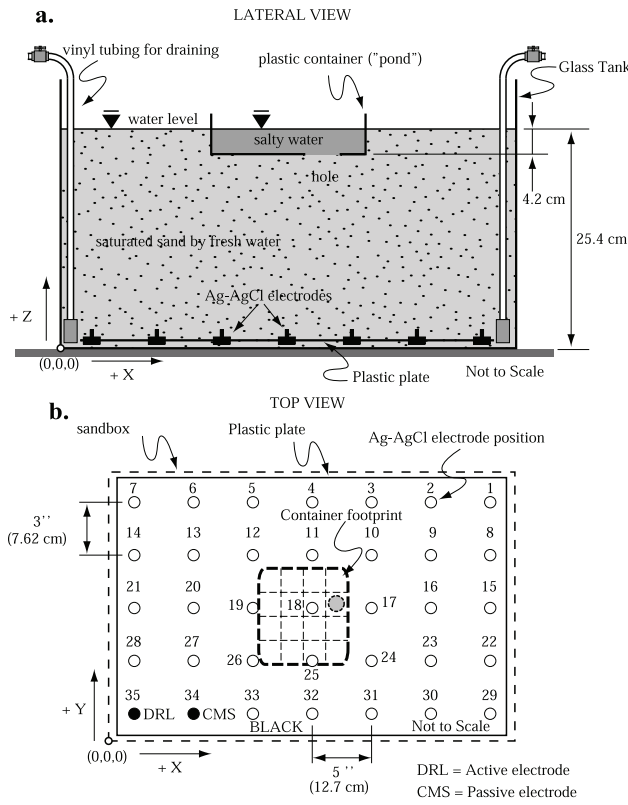
Figure 1. Sketch of the experimental setup. (a) Side view. (b) Top view with the position of the electrodes. CMS is the reference for the self-potential network of electrodes. The salt plume is sinking from the hole at the bottom of the container into the sandbox.

[5] Neglecting the effect of surface conductivity, the conductivity of the sand is,

$$\sigma = \frac{\sigma_f}{F}, \tag{7}$$

where $F = \phi^{-m}$ [–] is the electrical formation factor, σ_f is the pore water conductivity, and m [–] is the cementation exponent. The effect of the salinity upon the electrical conductivity is accounted for by using *Sen and Goode*'s [1992] model, which is valid from dilute concentrations to saturation in salt.

[6] In the widely used Fickian model, the hydrodynamic dispersion tensor is given by

$$\mathbf{D} = \left[\frac{D_m}{\alpha} + \alpha_T v\right]\mathbf{I}_3 + \frac{\alpha_L - \alpha_T}{v}\mathbf{v} \otimes \mathbf{v}, \tag{8}$$

where D_m is the molecular (mutual) diffusion coefficient of the salt (in $m^2\,s^{-1}$) (for a NaCl solution, D_m is ~1.60×10^{-9} m^2 s^{-1} at infinite dilution and ~1.44×10^{-9} m^2 s^{-1} at high salinities at 25°C), $v = |\mathbf{v}|$, $\mathbf{a} \otimes \mathbf{b}$ represents the tensorial product between vectors $\mathbf{a}$ and $\mathbf{b}$, and α_L and α_T are the longitudinal (along $\mathbf{v}$) and transverse (normal to $\mathbf{v}$) dispersivities (in m),

and α is the tortuosity. A generalization of equation (8) to anisotropic media is given by *Neuman et al.* [1987].

3. Forward and Inverse Modeling

[7] The forward modeling of the field equations obtained by combining equations (1) to (6) and the additional relationships for the material properties can be done with the finite element solver Comsol Multiphysics 3.4. We first solve the coupled hydrodynamic equations including the buoyancy effect. Then the solution is used to solve for the divergence of the source current density in the electrostatic Poisson equation,

$$\nabla \cdot (\sigma \nabla \varphi) = \nabla \cdot \mathbf{j}_S. \tag{9}$$

$$\mathbf{j}_S = \bar{Q}_V \mathbf{u} - \frac{k_b T}{Fe}\left(2t_{(+)} - 1\right)\nabla \sigma_f, \tag{10}$$

where $t_{(+)}= 0.38$ (NaCl) is the microscopic Hittorf number of the cations in the pore water. Equation (10) is obtained using the approach outlined by *Revil* [1999].

[8] We are looking for an algorithm to invert $\mathbf{k}$ and $\mathbf{D}$ accounting for the resistivity distribution, which change over time, and the possible non-uniqueness of the inverse problem. We use below a variant of the Markov chain Monte Carlo methods called the Adaptive Metropolis Algorithm (AMA) [*Haario et al.*, 2001]. This algorithm is based on the traditional Metropolis algorithm with a symmetric Gaussian proposal distribution centered at the current model $\mathbf{m}^i$ and with the covariance $\mathbf{C}^i$ that changes during the sampling in such a way that the sampling efficiency increases over time. Recent applications of the AMA algorithm are given by *Jardani and Revil* [2009] for the joint inversion of temperature and self-potential data.

4. Experiment and Inversion

[9] We performed a sandbox experiment to invert the time-lapse self-potential response resulting from the leakage of salty water from a small tank into a sandbox (Figure 1). The sandbox was filled with a well-sorted silica sand with a measured porosity of 0.40, a measured cementation exponent $m = 1.5$, and a median grain diameter $d_{50} = 350$ μm. The sandbox was initially saturated with tap water with $\sigma_f = (1.5 \pm 0.2) \times 10^{-3}$ S m^{-1}). We can check that surface conductivity can be neglected as the ratio of surface to bulk conductivity Du $= 4\Sigma_S/d_{50}\sigma_f = 5 \times 10^{-2} \ll 1$ where $\Sigma_S(25°C) = 4 \times 10^{-9}$ S is the specific surface conductance.

[10] On the top of the saturated sand surface, we setup a small plastic container with an impervious and insulating (plastic) boundary (Figure 1). This container was filled with salty water (NaCl, ~5.5 Mol/L, $\sigma_f = 34.5$ S m^{-1} at 20°C). The level of water was the same in the container and in the sandbox. On the bottom of the container, a hole was made with a diameter of 1.25 cm (Figure 1). The hole was initially sealed with a sticky tape before starting the measurements. Prior to removing the tape, the self-potential signals were recorded over 193 s (Figure 2). In this phase (termed Phase I below), all of channels had values 0.0 ± 0.2 mV. The second measurement phase (Phase II) started with the opening of the hole by removing quickly the sticky tape. This resulted in the formation of a salty plume sinking in the tank. At the start
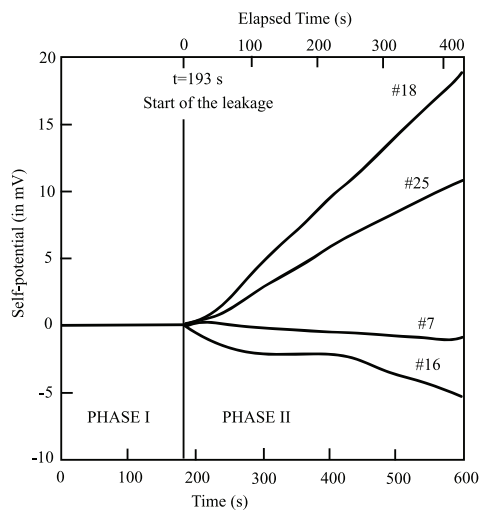
Figure 2. Display of the electrical potentials versus time for some selected electrodes (electrodes # 7, 16, 18, and 25), see position Figure 1 located at the bottom of the sandbox.

of Phase II, all the channels showed a significant and progressive change in their self-potential values (Figure 2). The highest changes were recorded approximately below the position of the hole (see Figure 3). Figure 3 shows maps of the self-potential distributions at total times 211 s, 303 s, 382 s, and 421 s (the leakage starts at $t = 193$ s). The experiment was run at 21°C.

[11] A network of 34 sintered 1 mm Ag/AgCl electrodes was located at the bottom of the sandbox (Figure 1). The electrodes were connected to a precision digital Biosemi voltmeter having a capacity of 256 sensor-signals digitized

with 24 bit resolution. Each channel consists of a low noise DC coupled post-amplifier, with a first order anti-aliasing filter, followed by a 24 bit Delta-Sigma modulator based analog to digital (AD) converter having an oversampling rate of 64, achieved with a post modulator decimation filter. The digital output of all the AD converters are digitally multiplexed into a serial data stream and sent to a PC via a single optical fiber through a fiber to USB interface. This voltmeter has a resolution of ∼0.1 μV (amplification of 32), a bandwidth from DC to 400 Hz, with an input impedance for the electrodes of 300 Mohm at 50 Hz [*Haas and Revil*, 2009].

[12] We use a 128 Hz acquisition sampling and later the data were further decimated in Matlab to a final decimated sample rate of 32 Hz. Because the data acquisition system was designed to have a broad dynamic range, electronic circuits were designed into it to minimize the effects of common mode signals appearing at the electrode inputs. These circuits employ 2 electrodes, the Common Mode Sense (CMS) and Dynamic Reference Level (DRL) electrodes. These electrodes are used together in a feedback loop to keep the potential of the sand close to the reference level of the AD converters used in the system. All electrode voltages measured are referenced to the CMS electrode. The DRL electrode is the only driven electrode in the measurement system and provides the signal return path for all of the electrodes with a current limit of 50 μA. These 2 electrodes form a dynamic reference level circuit that is used in the system to improve common mode voltage rejection.

[13] Forward modeling was performed with an open boundary at the top of the sandbox (except at the position of the small tank) and impervious boundaries elsewhere. The mass density of salt water at saturation is $\rho_S = 1202$ kg m^{-3}. The head at the bottom of the container is therefore $\rho_S\, g\, \delta h = 495$ Pa with $\delta h = 4.2$ cm. At the same depth the head in the
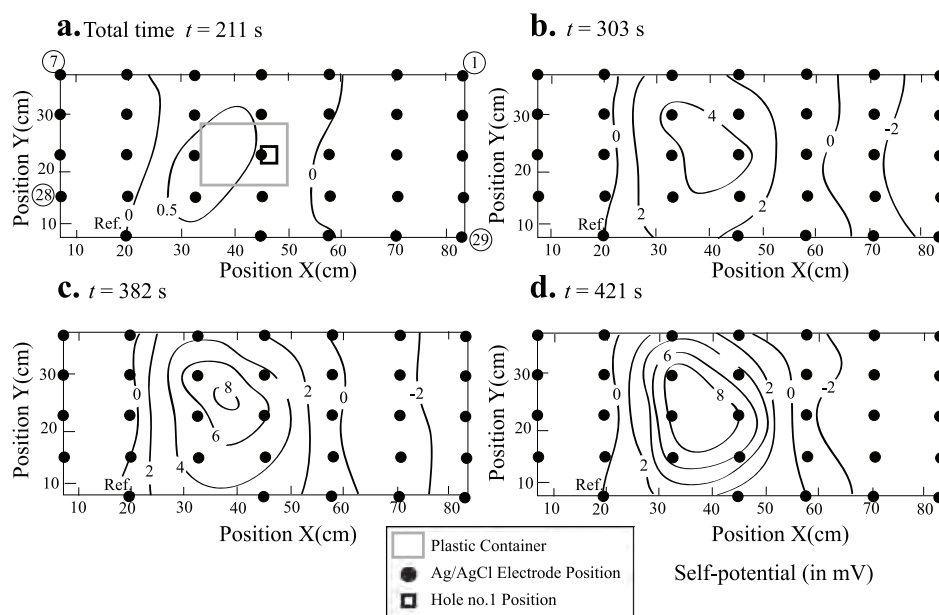
Figure 3. Self-potential maps at the bottom of the tank versus time at four different total times following the start of the leak at $t = 193$ s (Phase II). The distribution before $t = 193$ s is zero everywhere with an uncertainty of 0.2 mV. "Ref" indicates the position of the reference electrode (0 mV).
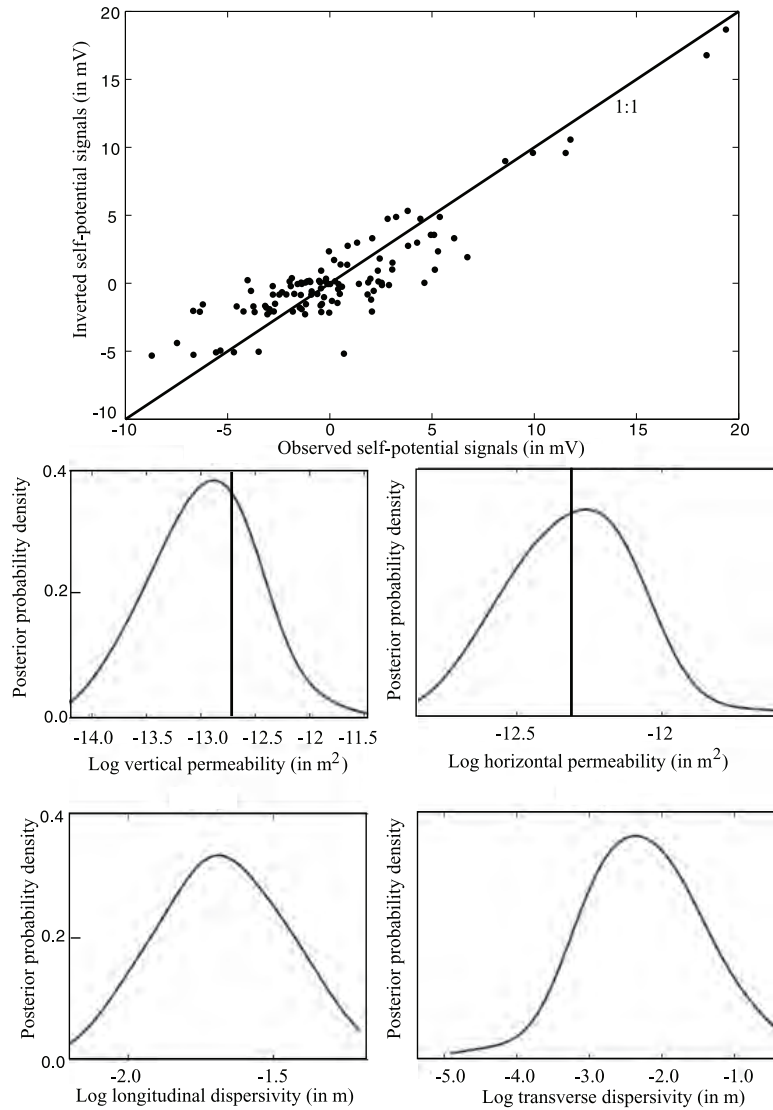
Figure 4. Posterior probability densities for the four model parameters and quality of the fit of the self-potential data using the most likely values of the four model parameters (RMS = 0.79). The model parameters include the two components of the permeability tensor and the transverse and longitudinal dispersivities. The vertical bars correspond to the measured vertical and horizontal permeabilities.

sandbox is ρ_w g δh = 412 Pa with ρ_w = 1000 kg m^{-3}. Therefore there is an excess head in the reservoir of 83 Pa, which represents an excess of head of 0.85 cm of fresh water. Knowing the size of the leaking area (a hole with a diameter of 1.25 cm), Q_S can be easily computed as a function of time. All the external boundaries are insulating.

[14] We consider that because the sand has angular grains, compaction of the sand in the sandbox is responsible for a vertical transversely isotropic material with the horizontal plane as the symmetry plane. Therefore in a Cartesian coordinate system with z the vertical axis, the permeability tensor is given by

$$\mathbf{k} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_x & 0 \\ 0 & 0 & k_z \end{bmatrix}. \tag{11}$$

[15] The model vector is $\boldsymbol{m}$ = (log k_x, log k_z, log α_T, log α_L) to ensure the positiveness of the four material properties. The likelihood function used to assess for the quality of a model $\mathbf{m}$ is:

$$P(\mathbf{d}|\mathbf{m}) = \frac{1}{\left[(2\pi)^N \det \boldsymbol{C_d} \right]^{1/2}} \exp\left[-\frac{1}{2} (g(\mathbf{m}) - \mathbf{d})^T \boldsymbol{C_d}^{-1} (g(\mathbf{m}) - \mathbf{d}) \right] \tag{12}$$

where $g(\mathbf{m})$ is the forward modeling operator for the chemiohydroelectrical problem. It connects the generation of a self-potential anomaly to a variation of the material properties, $\mathbf{d}$ is a N-vector of the observed self-potential data at different time intervals. The ($N \times N$)-covariance matrix $\boldsymbol{C_d}$ is a diagonal matrix determined using the standard deviations on the measurements (1 mV). We use the AMA algorithm to explore the posterior probability density $\pi(\mathbf{m}|\mathbf{d})$

of the model parameters **m** given the data **d**, $\pi(\mathbf{m}|\mathbf{d}) \propto P(\mathbf{d}|\mathbf{m})P_0(\mathbf{m})$ with $P_0(\mathbf{m})$ the prior probability density or belief on model parameters **m** also taken Gaussian distributed. The Bayesian solution of the inverse problem is the whole posterior probability distribution of the material properties. Let us assume that we have sampled the states $(\mathbf{m}^0, \ldots, \mathbf{m}^{i-1})$ where $\mathbf{m}^0$ corresponds to the initial state. Then a candidate point $\mathbf{m}'$ is sampled from the Gaussian proposal distribution q with mean point at the present point $\mathbf{m}^{i-1}$ and with the covariance:

$$C^i = \begin{cases} C^0 & \text{if } i \leq n_0 \\ s_n K^i + s_n \varepsilon I_n & \text{if } i > n_0 \end{cases}, \qquad (13)$$

where I_n denotes the n-dimensional identity matrix, $n_0 = 4$ model parameters, $\varepsilon = 10^{-16}$, $\mathbf{K}^i = \text{Cov}(\mathbf{m}^0, \ldots, \mathbf{m}^{i-1})$ is the regularization factor (a small positive number that prevents the covariance matrix from becoming singular), $\mathbf{C}^0$ is the initial covariance matrix, $s_n = (2.4)^2/n$ is a parameter that depends only on the dimension of the vector $\boldsymbol{m} \in \Re^n$ [*Gelman et al.*, 1996]. The candidate realization $\mathbf{m}'$ is accepted with the acceptance probability:

$$\alpha\left(\mathbf{m}^{i-1}; \mathbf{m}'\right) = \min\left[1, \frac{\pi(\mathbf{m}'|\mathbf{d})}{\pi(\mathbf{m}^{i-1}|\mathbf{d})}\right]. \qquad (14)$$

[16] If the candidate realization is accepted, we consider $\mathbf{m}^i = \mathbf{m}'$, otherwise $\mathbf{m}^i = \mathbf{m}^{i-1}$. The AMA algorithm was used to generate 10,000 realizations. We use the self-potential maps at the following times only: 211, 303, 421, and 600 s. We use the constraints: $\log(k_x, k_z) \in [-11; -14]$ and $\log(\alpha_T, \alpha_L) \in [-5; 0.5]$ with a uniform prior distribution on these intervals. After convergence, the statistics on the distribution of the model parameters are described by their posterior probability densities determined using the last 3000 realizations (Figure 4).

[17] The mean values of the inverted horizontal permeability agree well with the measured value in a sand permeameter ($\log k_x = -12.3$). The vertical permeability is found to be smaller in agreement with an independently measured value ($\log k_z = -12.7$). The result of the inversion for the longitudinal and transverse dispersivities are $\log \alpha_L = -1.7 \pm 0.5$ and $\log \alpha_T = -2.3 \pm 1.0$. This is fairly consistent with the value reported by *Sato et al.* [2003] with $\log \alpha_L$ between -2.5 and -2.7 for glass bead with $d_{50} = 300 \ \mu$m. In unconsolidated sands, the dispersivity can be described as $\mathbf{D} = D\mathbf{I}_3$ with $D/D_m = 1/\alpha + 0.5 \ \text{Pe}^{1.2}$ [*Pfannkuch*, 1963; *Maineult et al.*, 2005] where α is the tortuosity of the pore space ($\alpha = F \phi = 1.6$) and $\text{Pe} = d_{50} v/D_m$ is the Peclet number. The velocity v of the front can be determined by the time taken by the salt plume to go from the small container to the electrodes (a strong electrical effect is created when the salt reached the electrode #18 at 1308 s). This time lapse is 1115 s. As the traveled distance is 21 cm, the velocity is therefore $v = 1.9 \times 10^{-4} \ \text{m s}^{-1}$. Using $d_{50} = 350 \ \mu$m, we get $\text{Pe} = 46$ and a dispersivity $D \sim 7 \times 10^{-8} \ \text{m}^2 \ \text{s}^{-1}$. The dispersivity can be compared with the inverted result using the self-potential

data: $\alpha_L v = 4 \times 10^{-6} \ \text{m}^2 \ \text{s}^{-1}$. Therefore, our estimates are fairly consistent with the properties of the sand.

5.   Conclusions

[18] This is the first time that time-lapse self-potential monitoring is used to invert the permeability and effective dispersities of a porous material. The advantages of the Bayesian approach coupled with a McMC solver are: (1) it accounts for the possible non-uniqueness in the solution of the inverse problem, (2) electrical resistivity changes over time is automatically accounted for, (3) the geometry of the sandbox and boundary conditions are accounted for. While we have dealt with an homogeneous sandbox, the present approach can be used for heterogeneous field. Such a task would be however difficult if the only source of information is self-potential because of the low resolution of potential field methods. It would be complementary to complex resistivity time-lapse measurements. In addition, the measured signals are very small so the application of this method in the field would require a very sensitive equipment and a correction of the temperature drift of the electrodes.

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